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Development of Dissolved Oxygen Model for a Highly Variable Flow River: A Case Study of Ravi River in Pakistan

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Abstract A framework for dissolved oxygen (DO) modeling of the Ravi River has been developed based on a combination of laboratory measurements and field and monitoring data. Both the classical Streeter-Phelps (CSP) and the modified Streeter-Phelps (MSP) models are used for DO simulations. The MSP model considers the carbonaceous biochemical oxygen demand (CBOD) and nitrogenous biochemical oxygen demand (NBOD) separately, whereas the CSP model is evaluated considering only the CBOD and NBOD is incorporated in the overall BOD utilization rate. CBOD, NBOD and BOD rates have been determined through long-term BOD analysis of five main wastewater outfalls and two surface drains discharging into the Ravi River over a 98 km stretch. Analysis by Thomas Method manifests strong coefficient of determination "R^2" between 0.72 and 0.98 for all the three types of BOD rates. Sensitivity analyses have also been carried out to find out a suitable reaeration rate formula for highly variable flows in the Ravi River. The CSP model results based on classical approach of considering only CBOD show significant difference between the model predictions and field measurements suggesting that NBOD needs to be incorporated for the model development. The dissolved oxygen values calculated using the MSP model and the CSP model based on overall BOD rate are in close agreement with field measurements and are thus suitable to model DO levels in the Ravi River.

Keywords Dissolved oxygen modeling · Highly variable flow · Hydrodynamic model · Rivers · Deoxygenation rates · Water quality management

1 Introduction

The Ravi River is the most polluted river in Pakistan. It receives untreated domestic and industrial wastewaters through five outfalls and two natural surface drains located between a stretch of 98 km between Ravi Siphon and Balloki Headworks (Fig. 1). As a result, anaerobic conditions prevail in a reach of about 60 km of the river under low flow conditions.

The flow in the river is highly variable from less than 10 to 10,000 m^3/s (Fig. 2). The low flow conditions prevail for a period of about 8 months within a year while flood conditions occur during monsoon season (Fig. 3). Due to these extreme flow variations and river morphology, river cross-sections have developed in a unique way to give rise to two different hydrodynamic conditions (Fig. 4).

A number of monitoring studies have been carried out in the past to assess the water quality of the Ravi River [1–5]. These studies have also been used to suggest the required degree of treatment for the wastewaters from the city of Lahore in order to maintain healthy dissolved oxygen (DO) levels in the river [6–10].

Water regulatory agencies have not adopted any of the remedies suggested in the above mentioned studies, and the river water quality has continuously been deteriorating due to increase in wastewater volumes every year as a result of population growth and industrial development. The objective of this paper is to propose a framework that can be used to develop a DO model for the Ravi River through a combination of laboratory measurements, field and monitoring data, and sensitivity analysis with respect to model parameters.

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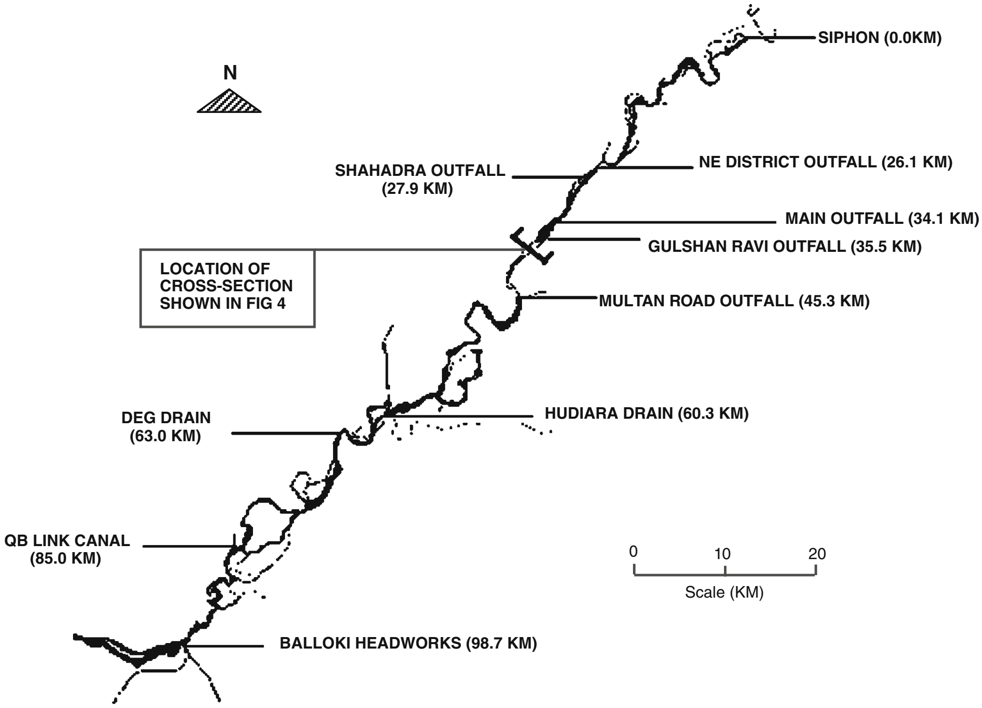


Fig. 1 Ravi river study reach from Ravi Siphon to Balloki Headworks

2 Methodology

The overall framework of the DO modeling of Ravi River is illustrated in Fig. 5. The main elements of the approach are river segmentation, the hydrodynamic model, and the water quality model. The river is segmented into different reaches based on variation in geometric properties (i.e., width and depth) and location of wastewater outfalls, surface drains and freshwater sources. Dissolved oxygen modeling requires simulating the hydrodynamics along with water quality. For this purpose, hydrodynamic model is developed for each reach using flow and river cross-section data. A water quality model is then developed with wastewater characteristics as the inputs, while the reaction rate coefficients have been established based on long-term carbonaceous biochemical oxygen demand (CBOD) and nitrogenous biochemical oxygen demand (NBOD) laboratory measurements. Knowing all the inputs and systems parameters of the selected DO model, the simulation is carried out to calibrate with field measurements under appropriate river flow conditions. The calibrated model then can be used to calculate the DO levels in the river at different levels of wastewater treatment. The degree of wastewater treatment that meets the desired DO standards in the river will establish the standards for wastewater treatment. The details of each aspect are given in the following sections.

2.1 River Segmentation

The major sources of pollution and water inflows to the River Ravi occur at a number of locations along the

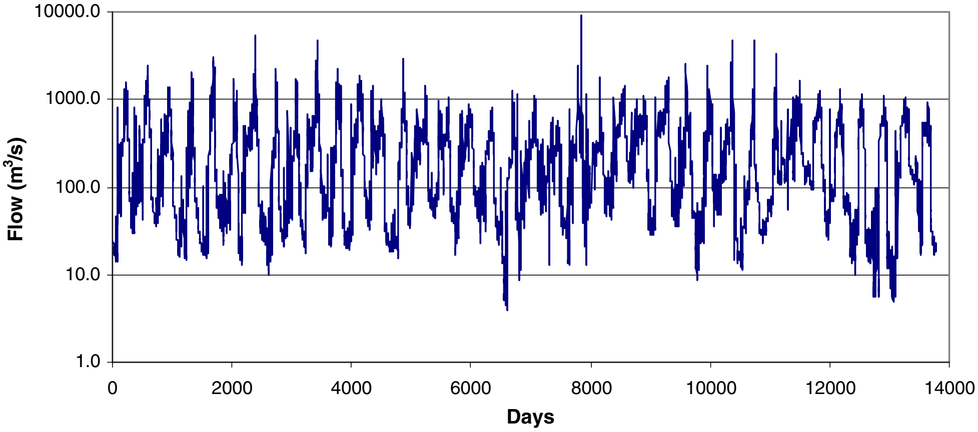
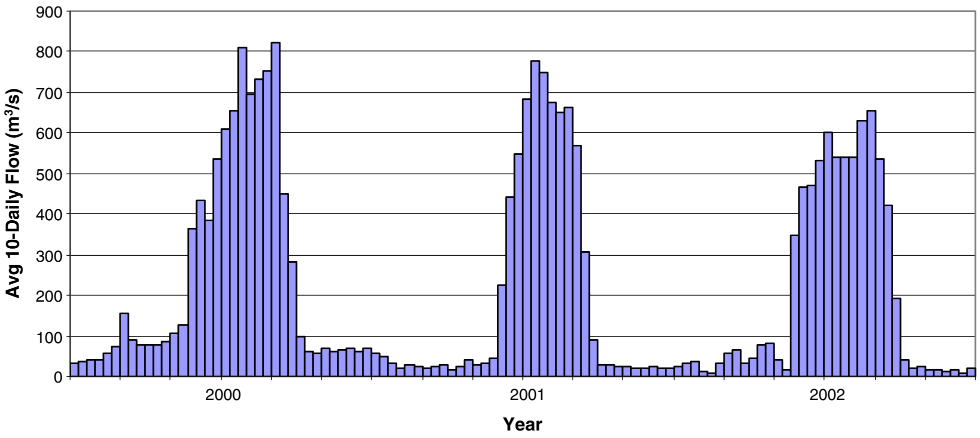


Fig. 2 Daily flow variations in Ravi river at Shahadra gauging station (developed using Irrigation and Power Department, Punjab data)

Fig. 3 Average 10-day flow variations for years 2001–2003 at Shahadra gauging station (developed by using Irrigation and Power Department, Punjab data)



selected river stretch between Siphon and Balloki Headworks, which are located between 846800N and 3355200E and 788400N and 3300500E, respectively (Fig. 1). The study length of the river is further subdivided into nine reaches based on the location of wastewater outfalls, surface drains, and freshwater tributaries (Fig. 6). The number of locations for which river cross-section data are available have also been shown in Fig. 6.

2.2 Hydrodynamic Model

The hydrodynamic model is based on the gauge-discharge data collected from Discharge Division of Punjab Irrigation Department [11]. Power functions are commonly used to relate mean velocity, depth, and width to discharge for the development of hydrodynamic models [12]. These can be written in the following forms;

$$U = aQ^b \tag{1a}$$

$$H = cQ^d \tag{1b}$$

$$B = eQ^f \tag{1c}$$

where Q is the discharge in m^3/s, U is the average velocity in m/s, H is the mean depth in m, B is the top width in m, and a, b, c, d, e, and f are the empirical constants which were determined from the river flow data.

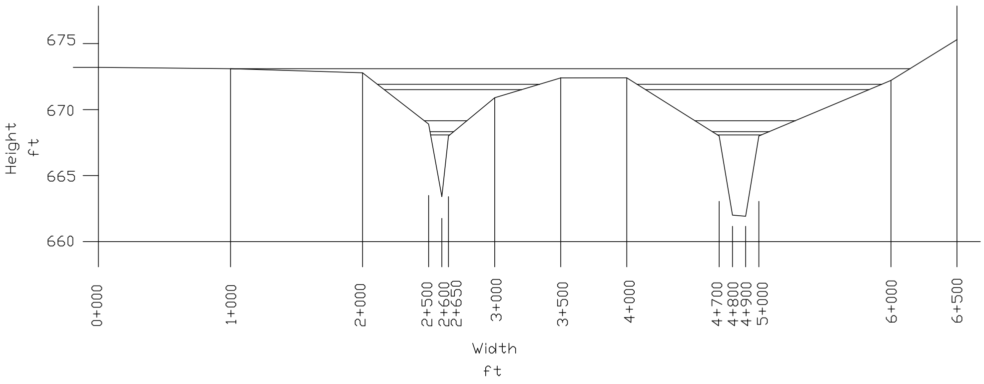
2.3 Wastewater Flows

Untreated domestic and industrial wastewaters are being discharged into Ravi River through five main outfalls and two large surface drains. The average wastewater flows of these disposal stations and drains collected from Water and Sanitation Agency [13], World Wildlife Federation, 2007 [14] and Irrigation Department and Power Department, 2004 have been used to estimate the current wastewater discharges into the river. Average BOD5 values worked out by Balfours Consulting Engineers [15] are used for the wastewater strength. These are given in Table 1.

3 Water Quality Model

Most stream DO models are based on the classical or modified form of Streeter-Phelps model [16]. The basic model includes only CBOD and reaeration. Various simple reaction equations, mixing, and sediment models have been

Fig. 4 A typical Ravi River cross-section (developed by data from survey division, Punjab Irrigation Department)



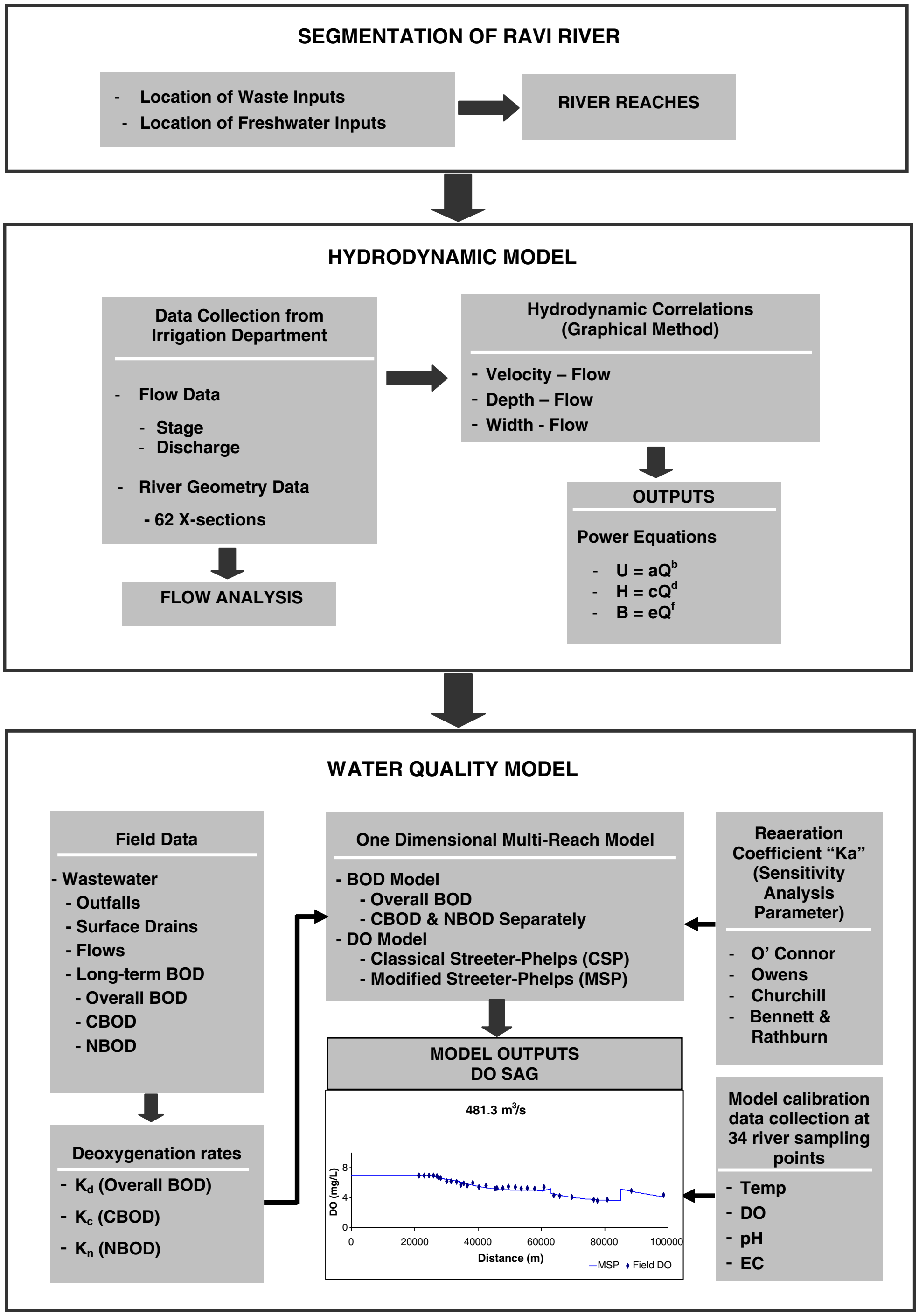


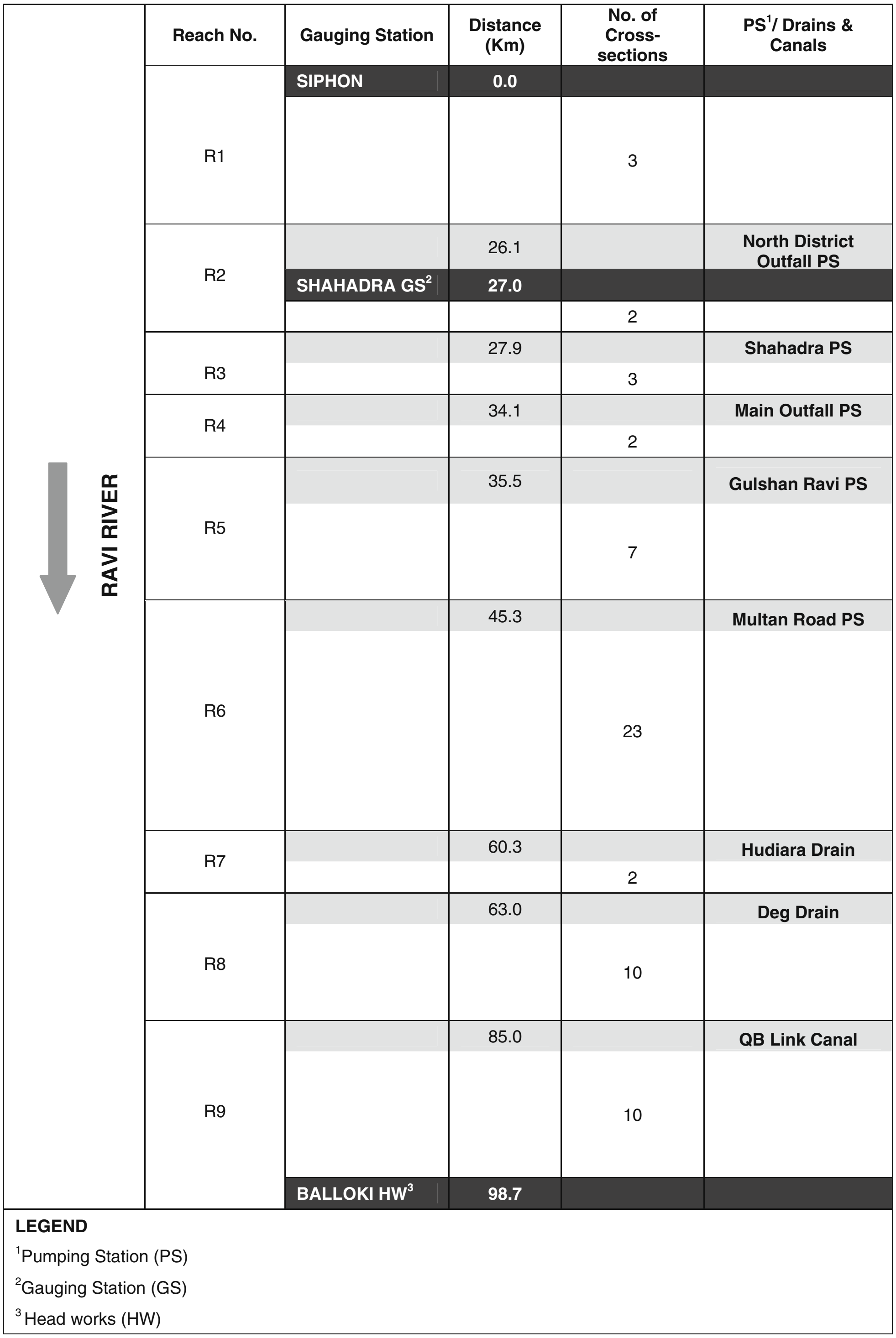
Fig. 5 Framework of water quality model for Ravi River

developed to supplement the extensions of Streeter-Phelps equations. Modified forms of the Streeter-Phelps model include other sinks of DO, i.e., nitrification, sediment oxygen demand, respiration by aquatic plants, and oxygen deficit from non-point sources and sources of DO such as photosynthesis by aquatic plants and DO from fresh water tributaries as depicted in Fig. 7 [17].

Changes in dissolved oxygen concentration (C) with all the sources and sinks can be presented in the form of a general mass balance equation in a segment as given below [18].

$$\begin{aligned} V(\mathrm{dC}/\mathrm{dt}) = & \text{reaeration} - \text{oxidation of CBOD} \\ & - \text{oxidation of NBOD} \\ & - \text{Sediment Oxygen Demand (SOD)} \\ & + \text{photosynthesis} - \text{respiration} \\ & - \text{oxygen transport by advection} \end{aligned} \qquad (2)$$

Fig. 6 Ravi River reach under study



Reach No.	Gauging Station	Distance (Km)	No. of Cross-sections	PS[1]/ Drains & Canals
R1	SIPHON	0.0		
			3	
R2		26.1		North District Outfall PS
	SHAHADRA GS[2]	27.0		
			2	
R3		27.9		Shahadra PS
			3	
R4		34.1		Main Outfall PS
			2	
R5		35.5		Gulshan Ravi PS
			7	
R6		45.3		Multan Road PS
			23	
R7		60.3		Hudiara Drain
			2	
R8		63.0		Deg Drain
			10	
R9		85.0		QB Link Canal
			10	
	BALLOKI HW[3]	98.7		

LEGEND

[1] Pumping Station (PS)

[2] Gauging Station (GS)

[3] Head works (HW)

where "V" is the volume of the segment. The mathematical form of eq. (2) can also be written as;

$$V\frac{\partial C}{\partial t} = K_a V(C_s - C) - \mathrm{VK}_d L - \mathrm{VK}_n L_n - \mathrm{VS} + \mathrm{PV} - \mathrm{RV} - U\frac{\partial C}{\partial x}V \quad (3)$$

where C_s is the DO concentration at saturation, C is the actual DO concentration in the river, K_d is the CBOD deoxygenation rate coefficient, L is the CBOD concentration, K_n is the nitrogenous deoxygenation rate coefficient, L_n is the NBOD concentration, P and R are the photosynthesis and respiration by the aquatic plants, respectively, S is the sediment oxygen demand, x is the distance in the river, and U is the river velocity.

Table 1 Flow rates and BOD concentration of the outfalls and the drains along Ravi River

S.No	Outfall/surface drains	Discharge (m^3/s)	BOD[a] (mg/L)
1	North District Outfall (PS)[b]	13.5[b]	285
2	Shahadra (PS)[b]	3.7[b]	230
3	Main Outfall (PS)[b]	13.4[b]	340
4	Gulshan Ravi (PS)[b]	9.0[b]	250
5	Multan Road (PS)[b]	4.5[b]	225
6	Hudiara Drain[c]	11.11[c]	130
7	Deg Drain[d]	91.4[d]	198
8	QB Link Canal[d]	544.0[d]	5

[a] Balfours

[b] WASA-LDA

[c] WWF

[d] Punjab Irrigation Department

The water quality management is usually done by determining degree of wastewater treatment at minimum average seven consecutive days flow in the river. So under these fixed flow conditions, volume "*V*" is constant and Eq. 3 becomes;

$$\frac{\partial C}{\partial t} = K_a(C_s - C) - K_d L - K_n L_n - S + P - R - U\frac{\partial C}{\partial x} \tag{4}$$

Equations 3 and 4 represent change in concentration of dissolved oxygen with time at a given location and variation along the distance. At steady state, the change in concentration at a given point with respect to time is zero, i.e., $\partial C/\partial t = 0$ and Eq. 4 becomes [19];

$$0 = K_a(C_s - C) - K_d L - K_n L_n - S + P - R - U\frac{dC}{dx} \tag{5}$$

or

$$U\frac{dC}{dx} = K_a(C_s - C) - K_d L - K_n L_n - S + P - R \tag{6}$$

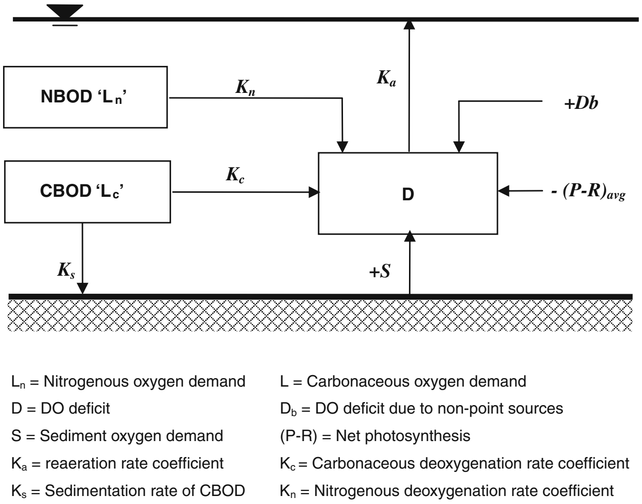


Fig. 7 DO sources and sinks in a surface water body (source: Schnoor, 1996)

Researchers and water quality engineers select the form of the model as per their requirement, availability of data, and processes of utmost importance. Generally, all the processes in dissolved oxygen balance do not occur simultaneously in a river. In case of the Ravi River photosynthesis and respiration are not expected due to high natural turbidity. In very fast moving rivers, sediment oxygen demand may not be an important parameter as sediments are frequently washed away with floods.

The classical Streeter-Phelps equation considers the deoxygenation through CBOD only in the river and reaeration as the only source of oxygen. Therefore Eq. 6 will become;

$$U\frac{dC}{dx} = K_a(C_s - C) - K_d L \tag{7}$$

To solve Eq. (7), it is common and convenient to express it in the form of DO deficit "*D*".

$$D = C_s - C \tag{8}$$

Thus aeration is a sink and BOD is a source in terms of oxygen deficit and differentiation of Eq. 8 results in;

$$\frac{dD}{dx} = -\frac{dC}{dx} \tag{9}$$

Therefore, Eq. 7 now can be written as;

$$U\frac{dD}{dx} = K_d L - K_a D \tag{10}$$

In a one-dimensional steady-state system, the solution to Eq. 10 in terms of DO deficit "*D*" can be written as [19];

$$D = D_0 e^{-K_a \frac{x}{U}} + \frac{K_d L_0}{K_a - K_d}\left(e^{-K_d \frac{x}{U}} - e^{-K_a \frac{x}{U}}\right) \tag{11}$$

where, D_o is the initial oxygen deficit and L_0 is the ultimate BOD in the river after mixing. The term "x/U" represents travel time "*t*" and Eq. 10 becomes;

$$D = D_0 e^{-K_a t} + \frac{K_d L_0}{K_a - K_d}\left(e^{-K_d t} - e^{-K_a t}\right) \tag{12}$$

The total oxygen utilization to complete the nitrification process is 4.57 g per gram of ammonia nitrogen [20]. The rate at which NBOD is exerted can be determined by long-term BOD analysis. Nitrification is an important process in the overall DO balance of a river. Both domestic and industrial wastewaters contain significant amounts of nitrogen, which needs to be considered in the DO modeling and management of any receiving water body. When both the CBOD and NBOD are considered separately, the modified Streeter-Phelps equation can be written as [19];

$$D = D_0 e^{-K_a t} + \frac{K_c L_{c0}}{K_a - K_c}\left(e^{-K_c t} - e^{-K_a t}\right) + \frac{K_n L_{n0}}{K_a - K_n}\left(e^{-K_n t} - e^{-K_a t}\right) \tag{13}$$

where, K_c is the carbonaceous deoxygenation coefficient, day^{-1}, K_n is the nitrogenous deoxygenation coefficient, day^{-1}, L_{c0} is the ultimate CBOD in the river after mixing of river water and wastewater, and L_{n0} is the ultimate NBOD in the river after mixing of river water and wastewater. DO deficit calculated using Eqs. 12 and 13 are brought into the terms of dissolved oxygen concentration using Eq. 8.

The BOD concentration and biodegradation are important inputs to DO model. The BOD calculations to use in DO model have been made by using a one-dimensional, multi-reach, steady-state model with first order BOD kinetics as [19];

$$L_c = L_{c0} e^{-K_c t} \tag{14}$$

where, L_c is the CBOD at any location in the river. The distance along the river is expressed as travel time "t".

A similar equation can also be used to model overall BOD including NBOD as;

$$L = L_0 e^{-K_d t} \tag{15}$$

where, L is the overall BOD at any distance (expressed as travel time "t") in the river and L_o is the ultimate overall BOD including NBOD.

The NBOD also follows a first-order kinetics like CBOD and therefore the mathematical equation representing variation of NBOD with travel time in the river can be written as [19];

$$L_n = L_{n0} e^{-K_n t} \tag{16}$$

where, L_n is the NBOD at any point in the river and L_{n0} is the ultimate NBOD at downstream of the point source entering into the river.

In this work, the classical Streeter-Phelps (CSP) model (12) was evaluated in two different ways to simulate dissolved oxygen concentration in the Ravi River. In the first case, only CBOD was considered whereas in the second case CBOD and NBOD are combined in the form of a single overall BOD rate equation. However, the modified Streeter-Phelps (MSP) model (13) was used to consider CBOD and NBOD separately.

4 Model Parameters

4.1 Deoxygenation Rates

BOD is the measure of organic matter present in the wastewater in terms of DO utilized to oxidize it. At the start of the BOD experiment, oxygen utilization starts for the oxidation of organic matter and therefore the value is zero. The organic content of the wastewater is reported by a standard BOD test that is conducted on a 5-day basis at 20°C and the results are reported as BOD5. The BOD values in Table 1 are BOD5 values collected from different sources and represent the strength of the wastewater. However, the oxidation of the organic matter continues, till is fully oxidized. In municipal wastewaters, most of the organic matter is typically oxidized in 20 days (i.e., long-term BOD analysis). Therefore, long-term BOD analysis of wastewater samples were carried out at five outfalls and two surface drains discharging into the Ravi River on grab samples to estimate rates at which oxygen is utilized (Fig. 8). All the samples were collected, prepared, and analyzed in accordance with Standard Methods for the Examination of Water and Wastewater [21].

Dissolved oxygen of the samples was measured using Standard Winkler method. Long-term BOD tests were conducted for BOD and CBOD by preparing two different sample sets using standard 300 mL bottles. In one set, nitrification was inhibited by adding 10 mg/L Nitrification Inhibitor, Formula 2533, Hach Co., Loveland (i.e., 2-chloro-6-trichloro methyl pyridine) in the dilution water. With each sample a set of blank and a set of GGA (glucose–glutamic acid) check were also analyzed for quality control and quality assurance. Overall BOD rate constant "K_d", CBOD rate constant "K_c" and NBOD rate constant "K_n" were determined by Thomas method [22].

The river deoxygenation rates K_d, K_c, and K_n are usually different from BOD bottle rates, because the BOD oxidation in a natural river includes phenomena which are not part of BOD bottle rate such as bio-sorption by biological slimes on river bottoms, river turbulence and roughness, and the density of attached organisms [18]. According to Wright and McDonnel [23], for large streams and rivers having flows greater than 800 cfs (22.7 m^3/s) river deoxygenation rates are similar to the bottle rates. Bosko [24] has proposed the following equation for estimating river deoxygenation rate from bottle rate constant as a function of river velocity and depth;

$$K_{d\,(\mathrm{River})} = K_{d\,(\mathrm{Bottle})} + \frac{U}{H}\eta \tag{17}$$

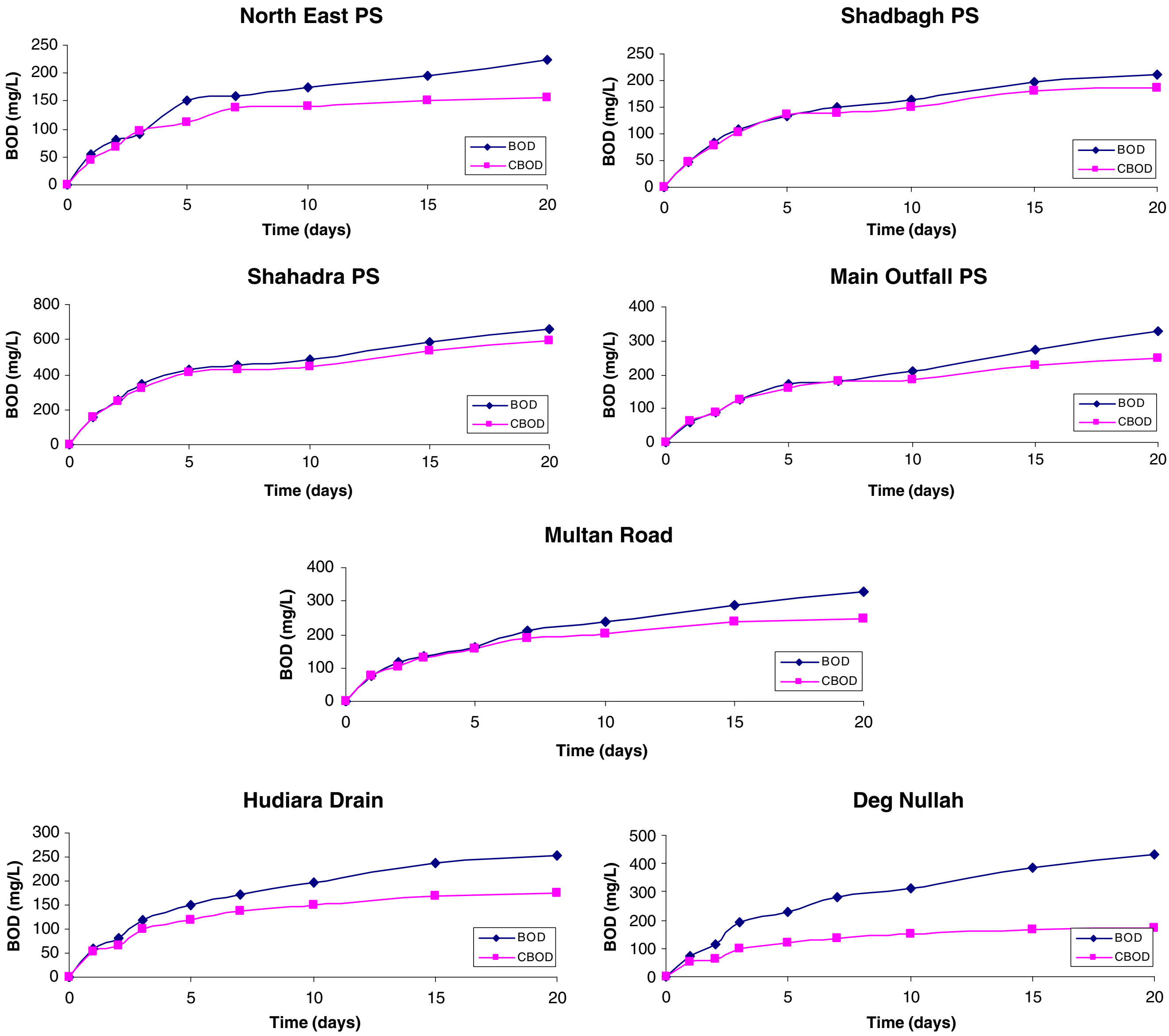


Fig. 8 Long-term BOD results at five wastewater outfalls and two surface drains along Ravi river

where, U is the average velocity of river in m/s, H is average depth of in m, and η is bed activity coefficient and its value ranges from 0.1 to 0.6 for stagnant or deep rivers to fast moving rivers, respectively. Average values for "U" and "H" of all the cross-sections in any segment of Ravi River have been used in Eq. 17. The temperature correction has been applied using the following equation;

$$(K_d)_T = (K_d)_{20}(\theta)^{T-20} \tag{18}$$

where $(K_d)_T$ and $(K_d)_{20}$ are decay coefficients at any temperature "T" and 20°C, respectively. In this work, "θ" was used as 1.047 for K_d and K_c, whereas a value of 1.08 was used for K_n determination [25].

4.2 Reaeration Rates

Atmospheric reaeration constant K_a is difficult to measure in the field and, therefore, empirical methods based on flow characteristics and depth are generally used. Empirical relationships developed by different researchers for large rivers are presented in Table 2. In all of these relationships, K_a is function of velocity "U" and depth "H". K_a values can be corrected for temperature using the following equation;

$$(K_a)_T = (K_a)_{20}(\theta)^{T-20} \tag{19}$$

$(K_a)_{20}$ and $(K_a)_T$ are the reaeration rate constants at 20°C and at any temperature "T" respectively. For the constant "θ", a value of 1.024 is used [26].

Table 2 River reaeration empirical formulas used in sensitivity analysis

S. no	Name of investigator	Formula	Parameters range
1	O'Connor-Dobbins [28]	$K_a = 3.93\frac{U^{0.5}}{H^{1.5}}$	H=0.3–9.14 U=0.15–0.49
2	Churchill et al. [29]	$K_a = 5.026\frac{U}{H^{1.67}}$	H=0.61–3.35 U=0.55–1.52
3	Owens and Gibbs [30]	$K_a = 5.32\frac{U^{0.67}}{H^{1.85}}$	H=0.12–0.73 U=0.3–0.55
4	Bennett and Rathburn [31]	$K_a = 5.5773\frac{U^{0.607}}{H^{1.689}}$	H=0.12–3.48 U=0.04–1.52

K_a reaeration rate constant (base e), day^{-1} ; U mean stream velocity, m/s; H mean stream depth, m

In the case of Ravi River where flow conditions are highly variable and river geometry behaves in two different power equations, it is considered appropriate to select K_a formula that is less sensitive to flow variations.

5 Field Measurements for Model Calibration

Under low-flow conditions, the study reach is devoid of oxygen, which shows that sufficient organic matter is still present to utilize the oxygen if made available. These levels cannot be estimated from the field measurements. Similarly, the data during high flows are of limited value as the DO sag may not develop. Therefore, intermediate flow conditions need to be assessed for collecting adequate data through field measurements for model calibration.

DO simulations at different flow conditions were carried out using the CSP and the MSP model with the hydrodynamic model of the river and the estimated parameters from the procedure given earlier. Due to the similar land use, values of deoxygenation rate coefficients determined for Multan Road outfall were also used for Gulshan Ravi outfall. Moreover, Nort-East and Shadbagh pumping stations joins the river through one outfall (i.e., North District Outfall), therefore the average of the results obtained from both of the pumping stations were used in DO simulations.

The model results in Fig. 9 show that under low flow conditions (147 m^3/s) the river becomes anaerobic at approximately 70 km downstream. At high flow conditions (1,982 m^3/s), DO levels are higher and a well-defined DO sag is not developed. At intermediate flow condition (425 m^3/s), a reasonable DO sag can develop, suggesting that flow in the river should be higher than 147 m^3/s and lower than 1,982 m^3/s to conduct a field survey for dissolved oxygen model calibration.

Examination of the past river flow data revealed that suitable flow conditions are expected in the months of June and July (Fig. 3). An extensive river survey was therefore conducted at a flow of 481.3 m^3/s (17,000 cfs) from 2nd to 5th of July 2008. Fortunately, no event of rainfall occurred during the whole field survey period. DO, pH, temperature, and total dissolved solids (TDS) measurements of the river water were determined at 34 locations along the 98 km length of the river under study. Temperature, pH, and TDS were measured with the help of HACH, portable meter, whereas DO was fixed at the site followed by their analysis with Winkler Method in the IEER laboratory according to the Standard Methods [21]. Global Positioning System (GPS, Magellan Explorist-400) was used to determine the exact distance between the sampling points along the river study reach. Northing and Easting noted from the GPS system were imported into the AutoCAD version 2006 through Pro-LINK Software.

6 Results and Discussion

The calibration of the Ravi River hydrodynamic model was done using the data of the survey division of the Irrigation and Power Department, Punjab. Since a single power function could not cover the entire flow range as indicated by the river geometry (Fig. 4), two different power equations, one for flows less than 283 m^3/s and the other for flows greater than 283 m^3/s were developed. Such correlations were developed for each reach. The sample results of Reach No.7 are given in Fig. 10. The estimated values of empirical constants of the power functions (a, b, c, d, e, and f) for all the reaches along with coefficient of determination (R^2) values are listed in Table 3. Strong correlations were observed for almost all the parameters of the power equations with R^2 values of generally more than 0.9. As all of the parameters (i.e., H, B, and U) are interrelated to each other, the coefficients of the power equations are not independent. Therefore, the sum of the exponents should be equal to unity ($b+d+f=1$)

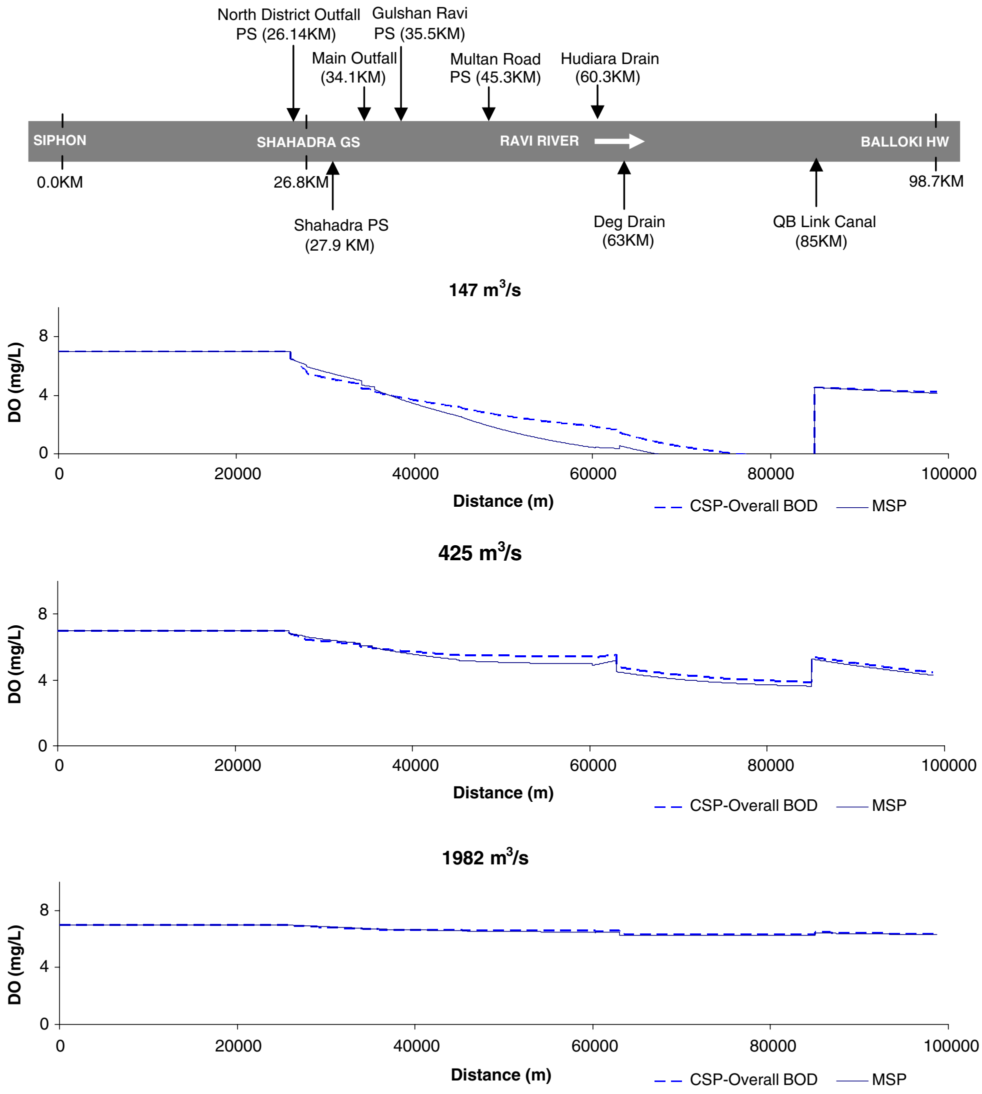


Fig. 9 DO model simulations for determining flow conditions for field measurements

[20]. The sum of the exponents b, d, and f estimated for the River Ravi hydrodynamic model were almost equal to “1” for most of the reaches. However, small difference from a value of 1 was found in some reaches that could be due to measurement errors and rounding off the values.

Sensitivity analysis for the reaeration rate relationships given in Table 2 have been carried out for the eight reaches (reach no. 2–9) of the river. Average velocities and depths were determined by using the power equations developed for each reach of the river. Variation of the reaeration rate coefficient with discharge as determined by different empirical relationships is reflected in Fig. 11a and b. The reaeration rates are up to 8 day^{-1} or even higher in reach no. 1–6 (Fig. 11a and b). The reaeration rate either decreases or does not increase significantly with flow in the last part of the river (reach nos. 7, 8, and 9) due to deeper and less wider cross-sections (Fig. 11b). However, in all of the reaches, the formula of O’Connor and Dobbins is least sensitive to variation in flows as reflected in Fig. 11.

The formula of O’Connor–Dobbins has been used for shallow and deep rivers (i.e., 0.3–9.14 m) for the last 50 years with acceptable results. For the Ravi River, average depth calculated by using hydrodynamic model for different reaches (Fig. 6) varies between 0.7 and 2.12 m under low flow to medium flow conditions (i.e., 147–425 m^3/s) with “K_a” values of 0.89–8.3 day^{-1}. The value of “K_a” ranges between 0.94 and 8.49 day^{-1} for the flow of 481.3 m^3/s at which the calibration survey was conducted. The results of sensitivity analyses (Fig. 11) and model calibration (Fig. 12) show that the “K_a” values determined

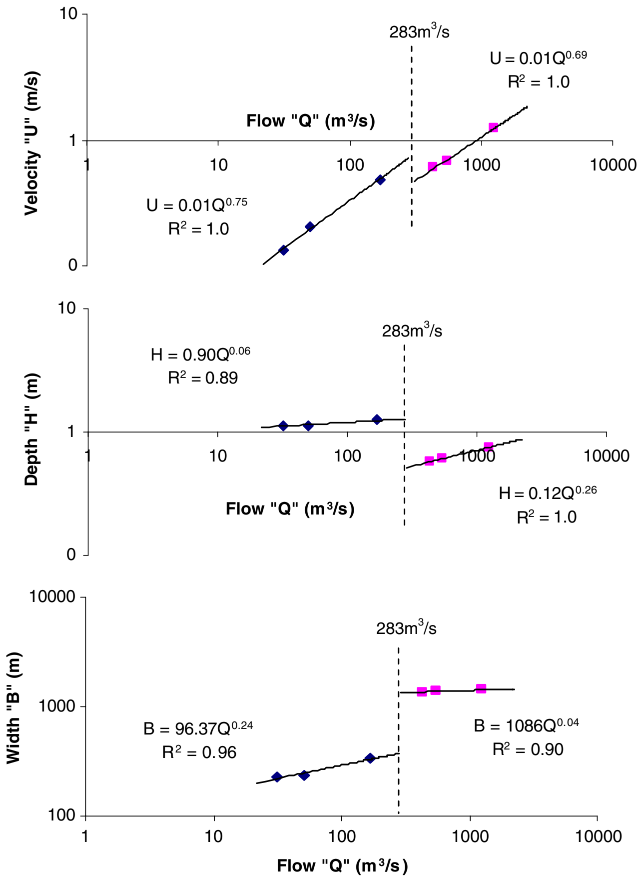


Fig. 10 Graphical presentation of the power functions for reach No. 7

for different segments of the river agree with the literature values of 0.05–12.2 day^{-1} [18].

The results of long-term BOD analysis of the outfalls and the surface drains are shown in Fig. 8. The ultimate BOD, CBOD, and NBOD (i.e., L_0, L_{c0}, and L_{n0}) and bottle biodegradation rates (i.e., K_d, K_c, and K_n) are unknown and need to be estimated. The first order BOD kinetics in terms of BOD remaining (L) can be written as [18–20];

$$\frac{dL}{dt} = -KL \tag{20}$$

the solution of the above Eq. 20 is;

$$L = L_0 e^{-Kt} \tag{21}$$

The BOD values shown in Fig. 8 determined from BOD experiments are BOD exerted values and are mathematically equal to;

$$y_t = L_0 - L \tag{22}$$

where, y_t is the BOD exerted. By putting value of "L" from Eq. 21 into the above Eq. 22, y_t can be calculated as;

$$y_t = L_0\left(1 - e^{-Kt}\right) \tag{23}$$

where, K is the bottle BOD rate and t is the incubation time in the bottle. Equation 23 is simply not form of a straight line and requires techniques to convert into a linear form. Thomas method is one of the commonly used methods for this purpose [22]. This method transforms the variables to result in a straight line where the time "t" is along x-axis and variable $(t/y_t)^{1/3}$ is along y-axis. The correlation between the variables can be judged by the coefficient of determination (R^2) [27].

Long-term BOD laboratory data generated for all the outfalls and drains for BOD (overall BOD including NBOD), CBOD, NBOD were analyzed with Thomas method to determine BOD bottle rate constants [22]. The results of long-term BOD analysis for determination of bottle rate constants and ultimate BOD values are presented in Table 4. Strong coefficient of determination (R^2) ranging between 0.91 and 0.98 are found both for CBOD and overall BOD rate constants. R^2 for K_n ranges between 0.72 and 0.93. The CBOD rate coefficient "K_c" ranges between 0.14 and 0.27 day^{-1} and the NBOD rate coefficient "K_n" ranges between 0.1 and 0.26 day^{-1}. These values are consistent with 0.1–0.5 day^{-1} reported in literature [18].

The results of laboratory analyses indicate that the wastewaters discharges from the city of Lahore exert significant NBOD (Table 4) suggesting that NBOD need to be considered for the DO modeling of the Ravi River.

The long-term BOD analyses of the surface drains (i.e., Hudiara Drain and Deg Drain) depict the progression of NBOD from the first day, which shows the presence of sufficient numbers of nitrifiers in the drain effluents (Fig. 8). The higher K_n value of 0.24 day^{-1} of Deg Drain compared to 0.09 day^{-1} of Hudiara Drain shows that most of the nitrogen has already been converted into NH_3-N (readily degradable form of nitrogen) before entering into the river (Table 4). This could be attributed to the completion of hydrolysis step in the drain due to long distance of about 20 km between the last wastewater input point in the drain and its confluence point with the Ravi River. Hudiara Drain, on the other hand receives industrial and municipal wastewaters continuously along its length up to about 4 km upstream of the confluence point with Ravi River. However, 90 mg/L of NBOD in Hudiara Drain is more than the Deg Drains with the NBOD value of 78 mg/L (Table 4).

Table 3 Estimated empirical constants for different reaches

Reach No.	*U*=aQ						*H*=cQ						*B*=eQ					
	Q^a<283 m^3/s			Q >283 m^3/s			Q<283 m^3/s			Q>283 m^3/s			Q<283 m^3/s			Q>283 m^3/s		
	a	b	R^2	a	b	R^2	c	d	R^2	c	d	R^2	e	f	R^2	e	f	R^2
1	0.01	0.79	1.00	18.28	−0.62	1.00	0.80	0.02	0.70	0.02	0.61	0.98	139.26	0.22	1.00	22.53	0.73	1.00
2	0.01	0.84	1.00	0.05	0.46	0.99	1.04	0.02	0.34	0.16	0.28	0.93	79.38	0.15	0.97	160.72	0.24	0.98
3	0.01	0.84	1.00	0.16	0.28	0.98	0.83	0.07	0.94	0.21	0.20	0.91	111.67	0.07	0.93	34.42	0.50	0.95
4	0.02	0.78	1.00	0.29	0.20	0.96	0.48	0.15	1.00	0.48	0.06	0.42	110.57	0.07	0.98	7.81	0.73	0.98
5	0.02	0.76	1.00	0.07	0.41	1.00	0.90	0.02	0.94	1.09	-0.03	0.14	47.49	0.31	0.98	23.63	0.58	0.99
6	0.02	0.73	1.00	0.05	0.45	0.96	0.92	0.03	0.91	0.48	0.07	0.99	35.18	0.43	0.95	90.34	0.40	0.92
7	0.01	0.75	1.00	0.01	0.69	1.00	0.90	0.06	0.89	0.12	0.26	1.00	96.37	0.24	0.96	1086.0	0.04	0.90
8	0.02	0.61	0.98	0.01	0.81	1.00	1.11	−0.04	0.90	0.65	0.04	0.99	73.43	0.43	0.94	490.48	0.11	1.00
9	0.002	0.80	1.00	0.001	0.98	1.00	1.29	0.09	0.94	2.18	−0.01	0.88	360.70	0.09	0.93	467.72	0.04	1.00

[a] Q is river flow in m^3/s

The river water quality data collected during the field survey is provided in Table 5. The maximum DO value is 7 mg/L (85% of the saturation value) upstream of North District pumping station before receiving the wastewater. The values drop gradually from 7 to 5.4 mg/L as moving downstream with the addition of wastewater flows from other pumping stations up to Deg Drain located at 63 km. There is a sudden drop in DO to 4.2 mg/L at the confluence of Deg Drain (63.9 km) due to large volume of wastewater (544 m^3/s), which decreases further to 3.3 mg/L at 80.8 km. The DO concentration improves to 4.6 mg/L when QB-Link Canal with freshwater inflows joins the river at 85 km.

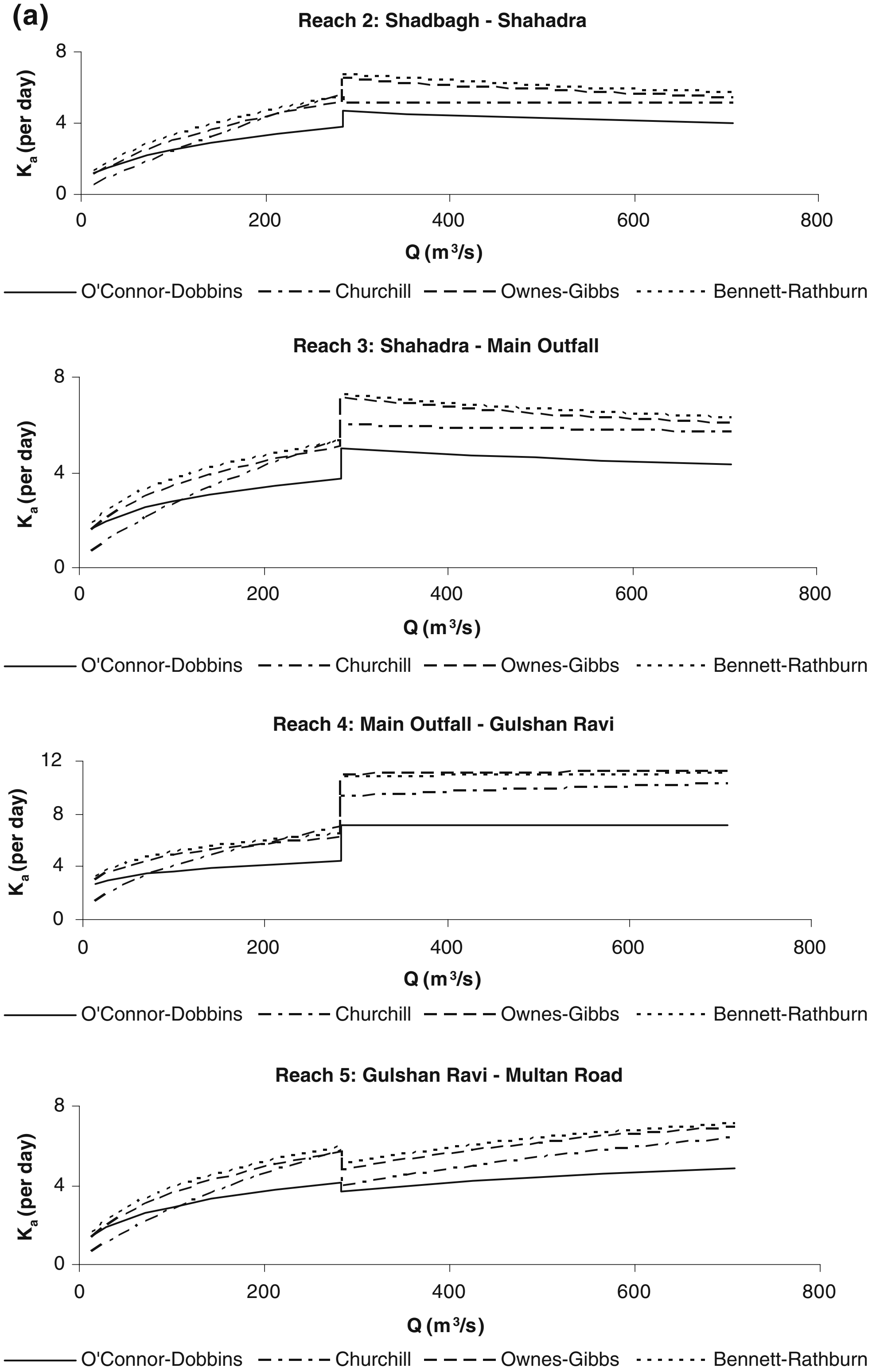
The TDS concentration in the river is 73 mg/L before receiving the wastewater and increases gradually to 86 mg/L until the confluence of Deg Drain, where a sudden increase to 110 mg/L is observed due to large industrial effluents in the drain water. The pH value however remains between 7.0 and 8.0 throughout the river stretch. There is a gradual raise in temperature up to 28°C in the last part of the river, the reason could be the afternoon time when atmospheric temperatures increase with time of the day during the month of July in Lahore. Temperature correction was nonetheless applied to the measured DO concentrations for model calibration.

The model results are compared with field measurements in Fig. 12. In the model calculations, η value of 0.17 was used for the first two reaches which are closely located to each other. For the rest of the reaches, bottle rates showed reasonable agreement with the field data and MSP calculation results. Lower values of "η" suggest that bottle rates may not need large adjustments as reported by previous researchers when the flows are greater than 22.7 m^3/s [24].

Comparison of model simulations with field measurements in Fig. 12a, demonstrates that when only CBOD is considered in CSP model, the calculated values are significantly different from the field measurements. The sum of square of the residuals (SSR) was 4.62 in this case. These results suggest that NBOD is important for DO simulation in the Ravi River and should be incorporated in model development.

The model results using CSP model with overall BOD and MSP model show reasonable agreement between the calculated values and field measurements. SSR values are 1.81 for CSP model and 1.23 for MSP model. Combining the CBOD and NBOD rates into an overall BOD rate is not a rational approach but can be used for initial water quality assessment of Ravi River where the constraints related to lack of resources exist. The development of present model is based on limited field studies. There is a need to collect water quality data under different flow conditions to fine tune the model parameters for its calibration. It would be

Fig. 11 **a** Sensitivity analysis for reaeration rate constant “K_a” reach Nos. 2–5. **b** Sensitivity analysis for reaeration rate constant “K_a” Reach Nos.6–9

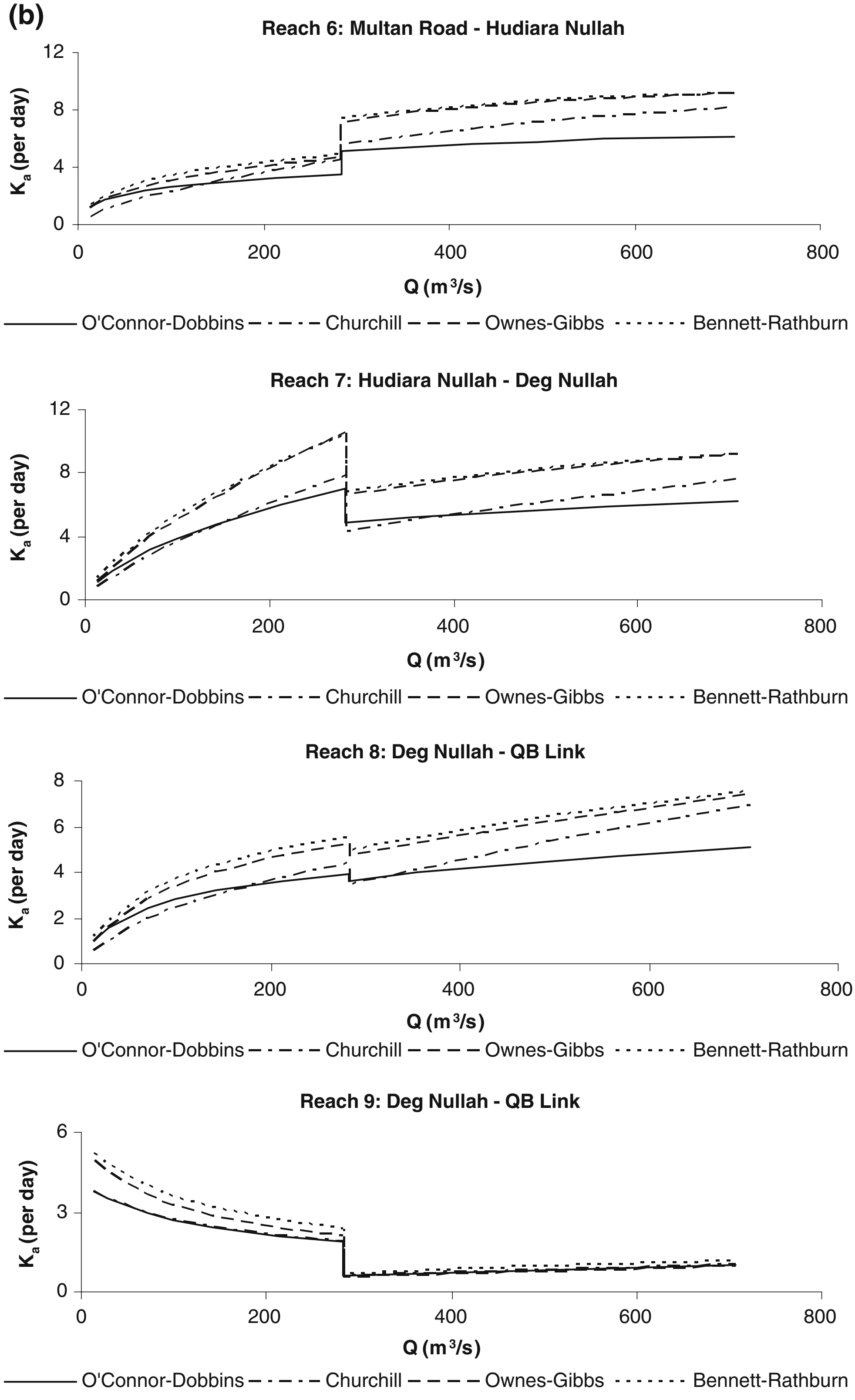


further desirable to verify the calibrated model under another set of data reflecting different river conditions. This would help to clearly reflect the different important processes including NBOD that are relevant to the Ravi River.

7 Conclusions

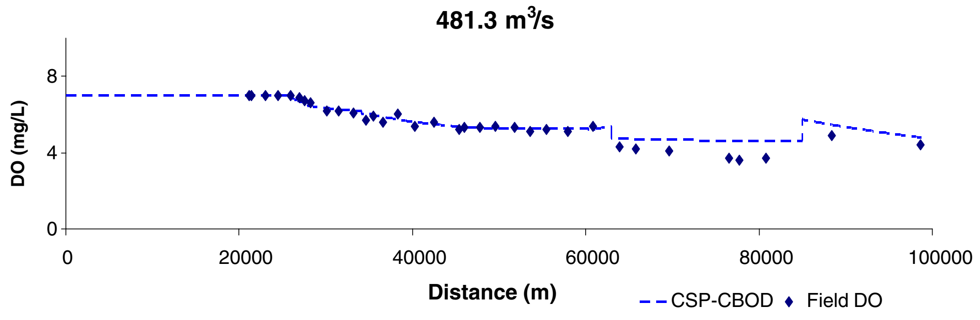
Dissolved oxygen modeling in rivers with highly variable flows and high pollution loads like that of the Ravi River in

Fig. 11 (continued)

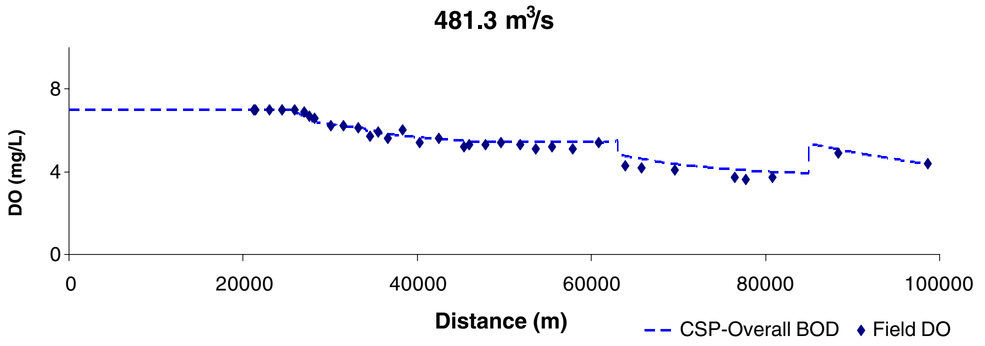


Pakistan is difficult to develop using field measurements due to anaerobic conditions under low flow and abrupt changes in river cross-sections when the flows are high. Therefore, a framework incorporating a combination of laboratory determinations, field measurements and sensitivity analysis is required for the development of DO model for the Ravi River. Model results when compared to DO measurements in the river water under intermediate flow conditions suggest that the

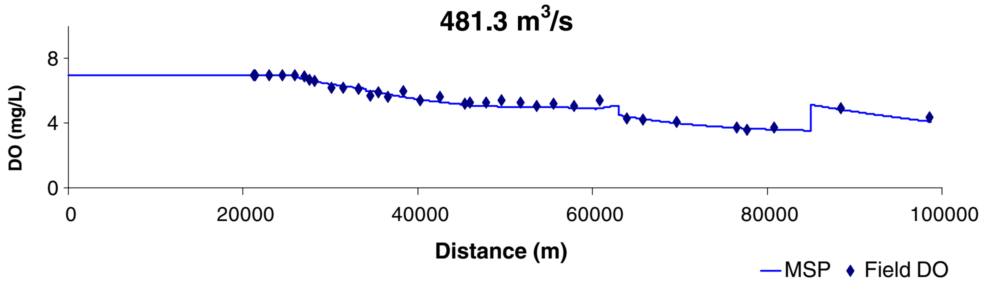
Fig. 12 Comparison of model results with field measurements



(a) CSP Model Considering Only CBOD



(b) CSP Model Considering Overall BOD



(c) MSP Model

Table 4 Long-term BOD analyses of wastewater outfalls and surface drains along Ravi River

Sr No	Wastewater Outfalls/ surface drains	BOD				CBOD				NBOD		
		BOD5 (mg/L)	BODu (mg/L)	K_d day^{-1}	R^2 TM[a]	CBOD5 (mg/L)	CBODu (mg/L)	K_c day^{-1}	R^2 TM[a]	NBOD (mg/L)	K_n day^{-1}	R^2 TM[a]
	Pumping stations											
1	North East	152	232	0.20	0.94	113	178	0.24	0.98	75	0.10	0.77
2	Shadbagh	134	224	0.21	0.97	137	202	0.23	0.97	24	0.26	0.88
3	Shahadra	391	676	0.20	0.94	413	622	0.23	0.95	69	0.23	0.92
4	Main Outfall	174	323	0.15	0.91	158	261	0.21	0.95	138	0.06	0.86
5	Multan Road	162	332	0.18	0.90	156	270	0.27	0.93	87	0.12	0.72
	Surface drains											
6	Deg Drain	228	450	0.16	0.96	175	380	0.14	0.95	78	0.24	0.93
7	Hudiara Drain	152	265	0.19	0.95	120	193	0.23	0.96	90	0.09	0.89

[a] *TM* Thomas method [22]

Table 5 Ravi River field measurement data

Sr no.	Distance down river from Siphon (km)	Field measurements			
		DO (mg/L)	TDS (mg/L)	pH	Temp (°C)
1	21.2	7.0	73.5	7.2	24.7
2	21.4	7.0	73.8	7.3	25.3
3	23.0	7.0	74.2	6.9	24.5
4	24.5	7.0	74.6	7.6	24.5
5	25.9	7.0	75.1	7.6	24.2
6	27.0	6.9	75.2	7.4	24.2
7	27.6	6.7	77.3	7.3	24.2
8	28.2	6.6	76.1	6.9	24.5
9	30.1	6.2	81.3	7.4	25.1
10	31.5	6.2	78.5	7.7	24.7
11	33.2	6.1	78.8	7.5	24.6
12	34.6	5.7	87.7	7.8	24.9
13	35.5	5.9	83	7.8	24.9
14	36.6	5.6	87.9	7.9	24.9
15	38.3	6.0	80.4	7.9	24.8
16	40.3	5.4	85.4	7.7	25.0
17	42.5	5.6	81.6	7.5	25.0
18	45.4	5.2	88.9	7.5	25.3
19	46.0	5.3	88.1	7.2	25.3
20	47.8	5.3	82.6	7.7	25.4
21	49.6	5.4	84.3	7.8	25.0
22	51.8	5.3	84.4	7.8	25.3
23	53.6	5.1	84.5	7.8	25.5
24	55.5	5.2	86.2	7.6	25.4
25	57.9	5.1	86.2	7.6	25.7
26	60.8	5.3	85.9	7.3	26.0
27	63.9	4.2	108	7.0	26.2
28	65.8	4.1	108.9	7.4	26.2
29	69.6	3.8	106.2	7.8	26.5
30	7.5	3.4	108.1	7.6	27.3
31	77.7	3.3	109	7.6	27.7
32	80.8	3.3	111.1	7.5	27.8
33	88.4	4.6	98	7.9	27.0
34	98.6	4.1	163	7.8	27.2

bottle rate constants determined through long-term BOD analysis can be used with some adjustments for large rivers like the Ravi River in the DO model.

O'Connor–Dobbins formula to calculate reaeration rate constant was found to be the least sensitive under variable flow conditions of the Ravi River based on sensitivity analysis. Its applicability in DO modeling of the Ravi River is reflected by a close agreement between the model results and the field measurements.

Classical Streeter-Phelps model considering only the CBOD as a sink of DO does not show reasonable agreement with field measurements thus highlighting the importance of NBOD for the Ravi River. The DO values calculated using modified Streeter-Phelps equation with separate CBOD and NBOD progressions shows close agreement with the field data, However, additional data will be require further calibration and verification of the model to develop strategies for water quality improvements in the Ravi River.

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