

# Mathematical modelling of stream DO–BOD accounting for settleable BOD and periodically varying BOD source

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## Abstract

The conventional Streeter and Phelps model does not account for the settleable component of BOD. Their model is therefore of little value in the present day context of polluted streams in which part of the BOD removal necessarily takes place through sedimentation, especially when untreated or partially treated wastes are discharged into streams. Several other dispersion models developed to date also do not account for the settleable part of BOD. In the work presented here, an attempt is made to present a mathematical model accounting for dispersion effects, settling of the settleable part of BOD and the periodic variation of the BOD source. The dispersion model takes into account the biofloculated sedimentation, as well as biochemical decay of nonsettleable BOD. An alternate finite difference scheme is used to solve the model representing the BOD–DO balance under the stated conditions in a stream. © 1999 Elsevier Science Ltd. All rights reserved.

*Keywords:* BOD; Sedimentation; Dispersion; Dissolved oxygen; Stream modelling

## Software availability

Various programmes have been developed in FORTRAN using Microsoft FORTRAN 77 version 3.20. They are readily available with first two authors.

## 1. Introduction

Natural waters are used for domestic, municipal, agricultural, irrigation, industrial, fisheries, recreation and other purposes. Such waters invariably get polluted with organic and inorganic chemicals, toxicants, pathogens, etc., originating from urban settlements, agricultural lands and industrial sources. Compared to the pollution caused by nature, anthropogenic pollution is of greater significance. It chiefly consists of organic waste. Organic matter discharged into the river utilizes the dissolved oxygen of the water through its aerobic decomposition. This depletes the dissolved oxygen in the water, causing problems for the survival of the biota (fish in particular). Dissolved oxygen is a surrogate variable for the general

health of an aquatic ecosystem. Monitoring of DO and the biochemical oxygen demand (BOD) provide adequate checks on the river conditions. Mathematical models are used as effective tools to predict DO conditions in the river. These mathematical models are differential equations representing the transport of BOD exerting organic matter and its impact on the river's DO. Existing mathematical models include the effects of advection, dispersion, decay of dissolved BOD, and reaeration from the atmosphere (Fischer et al., 1979; Koussis, 1983; Koussis et al., 1983, 1990; McBride and Rutherford, 1984).

The classical Streeter–Phelps (Streeter and Phelps, 1925) model is of little value as an accurate prediction of BOD and DO in a polluted stream as it does not account for BOD removal due to biofloculation (followed by sedimentation) which invariably takes place after the discharge of partially treated sewage into the streams. Velz and Gannon (1962) added a factor  $\alpha$  to the BOD rate constant,  $k_1$ , of the exponential form of a first-order kinetics to account for the BOD removal through sedimentation. But this is neither rational nor scientific as the settlement of particles cannot continue for an indefinite time. These shortcomings are removed

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by Bhargava (1983, 1986a & b). Bhargava considered that the total BOD ( $B$ ) of partially treated or untreated waste entering into the river consists of the settleable part ( $B_s$ ) and the dissolved parts ( $B_d$ ). It was assumed that the settleable part is removed at a very fast rate obeying the linear settling law (Bhargava, 1983).

The settleable and dissolved components of the BOD are to be determined analytically (Bhargava, 1983) for a given situation. This involves the use of Imhoff cones and would mainly depend on the strength and degree of treatment received by the wastewater entering into the river systems (Bhargava, 1983, 1986a).

The various dispersion models developed to date account for the BOD which is present entirely in the dissolved form and not the least in settleable form. They do not account for BOD removal due to bioflocculation (followed by sedimentation) which normally takes place after partially treated or untreated sewage outfalls drain into the stream. The benthic material, immediately after the outfall, undergoes anaerobic stabilization at the bottom. This effect is assumed to be negligible in the model. These models are therefore of little value for an accurate predication of BOD and DO after the sewage disposal points into the stream. In the present work, an attempt is made to find a numerical solution to the proposed model using an alternate finite difference scheme which is free from numerical dispersion.

## 2. Dispersion model accounting for sedimentation of BOD

The differential equations representing the transport of BOD and its impact on DO are developed using the following assumptions:

1. a one-dimensional model is applicable only after the mixing length is over;
2. the dispersion coefficient includes the effect of diffusion through longitudinal and turbulent mixing;
3. the entire BOD is not in the soluble form but a part of it is in the settleable form. The soluble part of BOD is decaying according to a first-order kinetics while the settleable part of BOD is being removed according to a linear law (Bhargava, 1983, 1986b). The ratio of settleable part to total BOD is assumed to be fixed at the outfall;
4. the BOD assimilation rate of the settleable component is too high as the removal of the settleable component takes less than an hour (the transition time) following a straight-line law. Over such short durations, the oxygen demands including those from the resuspension and resettling of the sedimented BOD shall be negligible and are therefore not included in the model developments;
5. for the settleable part of BOD diffusion is not being considered.

The size of settleable particles is bigger than the ionic size of the soluble BOD causing material. The settleable part of BOD would not, therefore, be transported through diffusion. The effect of advective forces has been considered and included in the transition time  $T_s$  (the time in which all the settleable part gets removed from the waste). It is greater than the mixing time.  $B_s$ , the settleable part of BOD, is removed by a linear settling law (Bhargava, 1983, 1986a):

$$B_s = B_{o-s} \left[ 1 - \frac{v}{d} \tau \right] \quad \tau \leq T_s \quad (1)$$

$$= 0 \quad \tau > T_s$$

where  $B_{o-s}$ , is the initial settleable BOD,  $v$  is the settling velocity of the particle and  $d$  is the depth of the stream.  $\tau$  is the time taken to travel a distance  $x$  from the source. The expression indicates that  $B_s$  would be completely removed within the transition time  $T_s = (d/v)$ . The transition time would be longer for deeper rivers. Since settling velocity depends upon the size of the particle, the transition time would be longer for smaller flocculated particles.

The validity of Eq. (1) is questionable when the source strength varies with time as in the case of malfunctioning of equipment or accidental spill of pollutant etc. The BOD concentration for the settleable part in such a situation would not be the same for all times at a particular distance. Since the concentration of settleable BOD at any time  $t$  at a particular distance would depend on the settleable BOD situated at the outfall  $\tau$  time earlier, the following expression for  $B_s(x,t)$  is suggested:

$$B_s(x,t) = B_{o-s}(0,t - \tau) \left[ 1 - \frac{v}{d} \tau \right] \quad \tau \leq T_s \quad (2)$$

$$= 0 \quad \tau > T_s$$

where  $\tau$  is the time taken to travel a distance  $x$  from the source.

Since the remaining part of the BOD is decaying exponentially with time, the transport equation for the dissolved part of BOD is given by

$$\frac{\partial B_d}{\partial t} + u(x) \frac{\partial B_d}{\partial x} = \frac{1}{A(x)} \frac{\partial}{\partial x} \left[ D_L(x) A(x) \frac{\partial B_d}{\partial x} \right] \quad (3)$$

$$- k_1 B_d$$

Considering the combined effect of  $B_s$  as well as  $B_d$ , the transport equation for DO is given by:

$$\frac{\partial C}{\partial t} + u(x) \frac{\partial C}{\partial x} = \frac{1}{A(x)} \frac{\partial}{\partial x} \left[ D_L(x) A(x) \frac{\partial C}{\partial x} \right] \quad (4)$$

$$- k_1 B_d - m B_s + k_r (C_s - C) \quad x > 0, t = 0$$

In Eqs. (3) and (4),  $B_d(x,t)$  is the non-settleable parts of BOD in mg/l;  $C(x,t)$  is the concentration of DO in mg/l at a distance  $x$  (m) downstream at a time  $t$  (s);  $u(x)$  is the mean cross-sectional flow velocity (m/s);  $D_L(x)$  is the coefficient of longitudinal dispersion ( $m^2/s$ );  $k_1$  is the decay rate of the dissolved part of BOD (i.e.  $B_d$ ) in ( $s^{-1}$ ) and  $m$  is the removal rate of the settleable part of BOD ( $s^{-1}$ ).  $k_r$  is the coefficient of reaeration ( $s^{-1}$ ) and  $C_s$  is the concentration of DO at saturation level mg/l. The value of  $m$  lies between  $k_1$  and  $k_r$ ,  $m \geq k_1$ .

### 3. Boundary conditions

The untreated or partially treated sewage is entering the stream through pipes. The pipes carry domestic sewage which is discharged into the stream. Since the strength of the domestic sewage depends upon the time of day, the source of the BOD is considered to be periodic with period  $2T = 24$  h. The following data is taken for the typical BOD of the stream at the outfall in a day.

The data in Table 1 represents the source strength of BOD at the outfall (at  $x = 0$ ) and can be represented as a Fourier Series:

$$B(0,t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi t}{T} + b_n \sin \frac{n\pi t}{T} \right) \quad (5)$$

where

$$a_0 = \frac{1}{2T} \int_0^{2T} B(t) dt$$

$$a_n = \frac{1}{T} \int_0^{2T} B(t) \cos \frac{n\pi t}{T} dt$$

$$b_n = \frac{1}{T} \int_0^{2T} B(t) \sin \frac{n\pi t}{T} dt$$

and

$$B(0,t + 2T) = B(0,t) \quad (6)$$

Fig. 1 shows the Fourier Series representation of the BOD data of Table 1.

It is also assumed that there is no other source of BOD downstream of the outfall except for the above stated source. The other initial and boundary conditions associated with Eqs. (3) and (4) are

$$B(x,0) = 0 \quad C(x,0) = C_s; \quad x > 0$$

$$C(0,t) = C_s; \quad t > 0 \quad (7)$$

and  $B(x,t) \rightarrow 0 \quad C(x,t) \rightarrow C_s$  as  $x \rightarrow \infty$

### 4. Method of solution

The transformations

$$D = C_s - C$$

$$B_d = b_d \exp(-k_1 t)$$

$$D = c \exp(-k_r t)$$

are used to eliminate unknown source and sink terms from Eqs. (3) and (4) giving Eqs. (8) and (9), respectively.

$$\frac{\partial b_d}{\partial t} + V(x) \frac{\partial b_d}{\partial x} = D_L(x) \frac{\partial^2 b_d}{\partial x^2} \quad (8)$$

$$\frac{\partial c}{\partial t} + V(x) \frac{\partial c}{\partial x} = D_L(x) \frac{\partial^2 c}{\partial x^2} + k_1 b_d \exp[-(k_1 - k_r)t] \quad (9)$$

$$+ m B_s \exp(k_r t)$$

where

$$V(x) = u(x) - \frac{D_L(x)}{A(x)} \frac{\partial}{\partial x} [A(x)] - \frac{\partial}{\partial x} D_L(x) \quad (10)$$

The above expression represents the effective velocity at a point due to combined action of advection and variable dispersion. The initial and boundary conditions to solve Eqs. (8) and (9) are accordingly transformed.

Eq. (8) is solved independently prior to the solution of Eq. (9). A procedure to solve Eq. (9) is presented here. A similar approach is used to solve Eq. (9). Eq. (9) is rewritten and split into two parts.

Table 1

Time (h)	0–4	4–6	6–8	8–12	12–16	16–18	18–20	20–24
Stream BOD (mg/l)	7	18	25	15	5	10	18	2

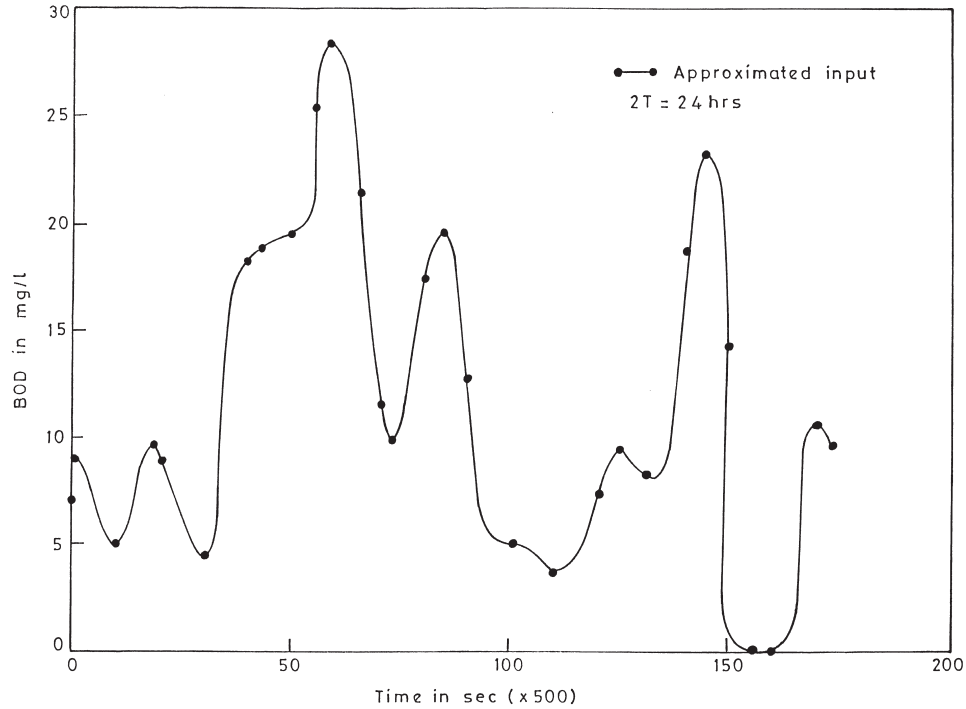


Fig. 1. The input BOD loading given by approximated Fourier series (taking only 13 terms).

$$\frac{1}{V(x)} \frac{\partial c}{\partial t} + \frac{\partial c}{\partial x} = \frac{k_1 b_d}{V(x)} \exp\{(k_r - k_1)t\} + \frac{mB_s}{V(x)} \exp(k_r t) \tag{11}$$

and

$$\frac{\partial c}{\partial t} = D_L(x) \frac{\partial^2 c}{\partial x^2} \tag{12}$$

The numerical solution of Eq. (11) is obtained by discretizing it over a nonuniform rectangular grid as follows:

$$\begin{aligned} & \frac{1}{2} \left[ \frac{c_{i+1,n+1} - c_{i,n+1}}{\delta x_i} + \frac{c_{i+1,n} - c_{i,n}}{\delta x_i} \right] \\ & + \frac{1}{V_i} \left[ \theta \left\{ \frac{c_{i,n+1} - c_{i,n}}{\delta t} \right\} \right. \\ & + \left. \left\{ (1 - \theta) \frac{c_{i+1,n+1} - c_{i+1,n}}{\delta t} \right\} \right] \\ & - \frac{k_1 b_{d_{i,n}}}{V_i} \exp\{(k_r - k_1)n\delta t\} - \frac{mB_{s_{i,n}}}{V_i} \exp\{k_r n\delta t\} = 0 \end{aligned} \tag{13}$$

In Eq. (13) the variable  $\delta x_i$  is the spatial grid distance computed as  $\delta x_1 = V_i \delta t$ ,  $V_i$  being the effective velocity at the  $i$ th grid. Rearrangement of Eq. (13) gives the

unconditionally stable explicit finite difference scheme as

$$c_{i+1,j+1} = A_1 c_{i,j} + A_2 c_{i,j+1} + A_3 c_{i+1,j} + A_4 b_{d_{i,j}} + A_5 B_{i,j} \tag{14}$$

and similarly for BOD

$$b_{d_{i+1,j+1}} = A_1 b_{d_{i,j}} + A_2 b_{d_{i,j+1}} + A_3 b_{d_{i+1,j}} \tag{15}$$

where,

$$A_1 = (1 + 2\theta)/(3 - 2\theta)$$

$$A_2 = A_3 = (1 - 2\theta)/(3 - 2\theta)$$

$$A_4 = 2k_1 \delta t \exp\{(k_r - k_1)j\delta t\}/(3 - 2\theta)$$

$$A_5 = 2m\delta t \exp(k_1 j\delta t)/(3 - 2\theta)$$

The values of  $b_{d_{i,j}}$  are computed from Eq. (15) while for computing  $B_{s_{i,j}}$  values, the expression given in Eq. (2) is used. Eq. (2) represents the  $B_s$  values in Lagrangian coordinates. But the  $B_s$  values required in the scheme (i.e. Eq. (14)) are computed in Eulerian coordinates. The following expression makes the two compatible.

$$B_{s_{i,j}} = B_{s_{0,1-j}} \left( 1 - \frac{v}{d} i \delta t \right) \quad i > j$$

and

$$B_{s_{i,j}} = 0 \quad i > j \tag{16}$$

The shaded region in Fig. 2 shows the grid where the settleable BOD will be nonzero and computed from Eq. (16).

Expansion of  $c_{i+1,n+1}$ ,  $c_{i,n+1}$  etc. by means of Taylor's theorem in terms of  $C_{i,n}$  upto the second order term and their substitution on Eq. (14) would yield

$$\frac{\partial c}{\partial t} + V(x) \frac{\partial c}{\partial x} = k_1 b_d \exp[(k_r - k_1)t] \tag{17}$$

$$+ m \cdot B_s \exp(k_r t) + \left[ \frac{V^2 x \delta t (1 - 2\theta)}{2} \right] \frac{\partial^2 c}{\partial x^2} + \dots$$

Comparison of Eq. (17) with Eq. (14) suggests the third nonzero term on the right hand side is a numerical dispersion error. It is observed that the parameter  $\theta$  representing the weighted average can be chosen in such a way that the coefficient of the third term on the RHS of Eq. (16) represents the physical dispersion coefficient  $D_L$  in a particular grid, i.e.

$$D_L = \frac{V^2 \delta t (1 - 2\theta)}{2}$$

This yields

$$\theta = 0.5 - \frac{D_L}{V^2 \delta t} = 0.5 - \frac{1}{p\#}; \quad 0 \leq \theta \leq 1 \tag{18}$$

Thus if  $\theta$  is given by Eq. (18) only the advection,

reaction Eq. (11) need be solved for the solution of Eq. (9). For a particular study the time interval  $\delta t$  is chosen in such a way that it may satisfy  $\delta t > 2D_L/V^2$ .

### 5. Results and discussion

To show the effect of sedimentation of BOD on DO, the following two cases are considered. The first case refers to the situation in which the entire BOD in the source is in dissolved form while in the second case a part of the total BOD is in settleable form. The total BOD in both cases is decaying exponentially with rate  $k_1$  with time at the outfall itself. The hydraulic parameters and stream configuration are described below.

The area of the river cross-section is assumed to vary linearly in the longitudinal direction as  $A(x) = A_0 + \alpha x$ , in which  $A_0$  is taken as 200 m<sup>2</sup> and  $\alpha = \pm 0.003$  m. The river flow rate  $Q$  ( $= 200$  m<sup>3</sup>/s) is assumed to be constant throughout the study reach. The stream velocity  $u(x)$  (m/s) is computed as  $u(x) = Q/A(x)$ .

The coefficient of dispersion  $D_L(x)$  is determined by the formula developed by Fischer et al. (1979) for natural streams,  $D_L(x) = [0.01u^2w^2/(u^*d)]$ , in which  $d$  is the depth (m) of the river assumed to be constant ( $= 4$ m),  $w$  is the width (m) varying linearly with  $x$  as  $w = 50 + 0.00075x$ .  $u^*$  is the friction velocity which is assumed to be 0.09 m/s. The  $k_1$ ,  $k_r$  and  $m$  values are assumed to be  $0.3 \times 10^{-4}$ ,  $0.6 \times 10^{-4}$  and  $0.6 \times 10^{-4} \text{ s}^{-1}$  respectively and  $B_0$  is assumed to be 28.0 mg/l. The DO in the stream upstream of the wastewater input point is assumed to be at a saturation level of 9.17 mg/l. The  $B_{o-s}$  and  $B_{o-d}$  values are assumed to be 16.0 and 12.0 mg/l, respectively.

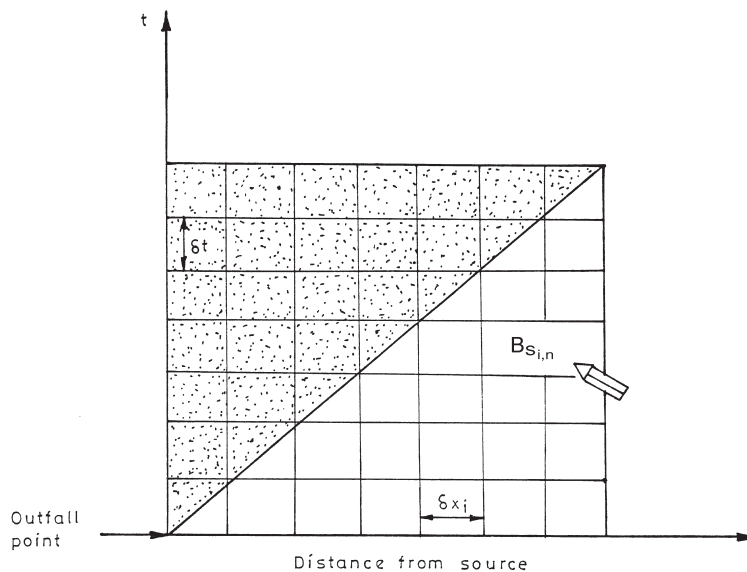


Fig. 2. Grid pattern of the numerical scheme.

Numerical solutions are first computed for  $i = 1$ , at  $x_1$ , using given conditions for  $t = n\delta t$ ;  $n > 1$ , until the steady state is reached. The solution is then advanced to  $x_{i+1} = x_i + \delta x_i$ ,  $i = 1, 2, 3, \dots$ . Thus obtained  $b_d$  and  $c$  values are transformed to  $B_d$  and  $D$  values with the help of transformations used earlier. Finally, the DO values are obtained by the relation  $C = C_s - D$ .

In Fig. 3 the concentration profiles of partly settleable BOD due to periodically varying BOD are drawn at a distance of 2.5 km. They are compared with the case when the BOD is entirely soluble. In Fig. 4, similar plots are drawn at a distance of 4.4 km. These distances lie inside the sedimentation region. Similar plots at distances 9.4 and 17.7 km are shown in Figs. 5 and 6, respectively. These distances are taken outside the sedimentation region.

It is observed from Figs. 3–6 that the variation of total BOD remains cyclic with the same period, however, the peak concentration is decreasing with distance downstream. The concentration of BOD at any distance for the case when part of the total BOD is settleable is less than the case when the entire BOD is soluble. This difference is remarkably high at the peak points. Since the settleable part is removed faster, it assimilates more biochemical oxygen demand and consequently the remaining total BOD would be less with subsequent distances.

Figs. 5 and 6 show the total BOD variation (after sedimentation is complete) with time. It is observed from the plots in Fig. 5 that the peak concentration for the case when part BOD is settleable is less than that when

the BOD is totally soluble, at a fixed distance. The reason is that when the entire BOD is in dissolved form then at any distance a higher concentration of BOD is remaining while in the other case the settleable part is finished within the transition time and only the remaining BOD is undergoing the process of advection dispersion and biochemical decay. The time to reach a particular distance is shown in both cases, and more BOD would remain at the distance in the case of entirely soluble BOD.

It is thus apparent, as expected, that the peak concentrations are more when the total BOD is entirely in a dissolved form as compared to a situation when the total BOD is in a settleable as well as nonsettleable (dissolved) form, and this is irrespective of the distance.

In Figs. 7 and 8, a comparison of DO variation with time for the above stated cases is shown at distances 2.5 and 4.4 km, respectively. Figs. 9 and 10 represent the variation in concentration of DO for the case when part of the BOD is settleable and for the totally dissolved BOD case at 9.4 and 17.7 km, respectively.

The reverse is happening for the dissolved oxygen at these distances shown in Figs. 7 and 8. It is observed from these figures that the DO remaining in the stream at 4.4 km is less than it was at 2.5 km. All other factors affecting the stream's DO being the same everywhere in the reach, the greater demand would consume more DO and consequently the DO would decrease with increased amount of BOD exertion. The plots in Figs. 3 and 4 adequately justify this aspect.

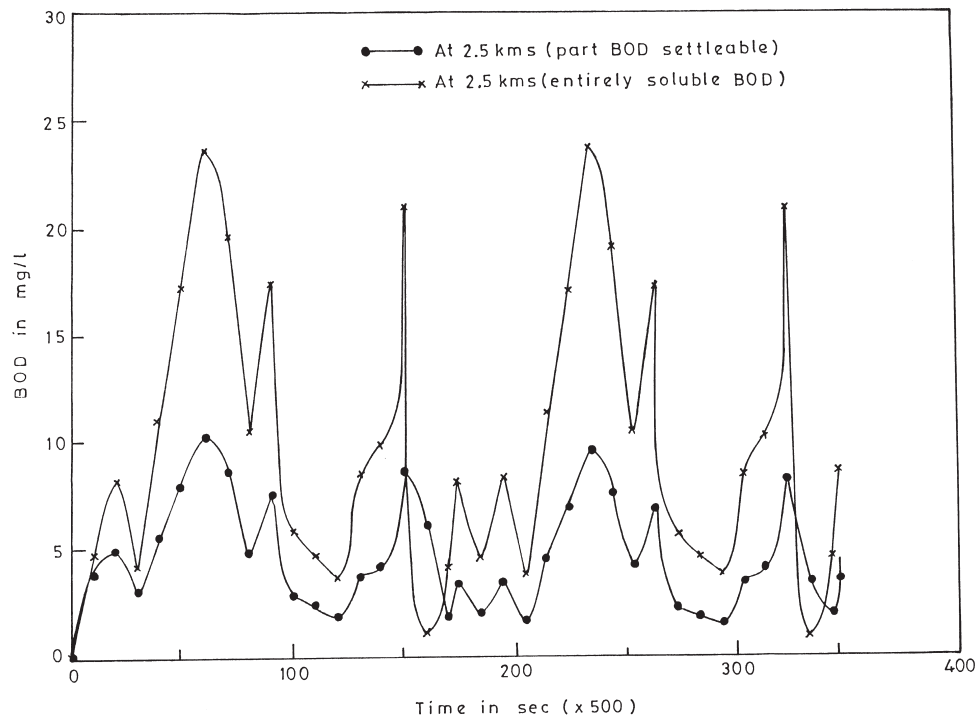


Fig. 3. Comparison of distribution of total BOD for the case when part BOD is settleable with the case of entirely soluble BOD at 2.5 km (inside the sedimentation region).



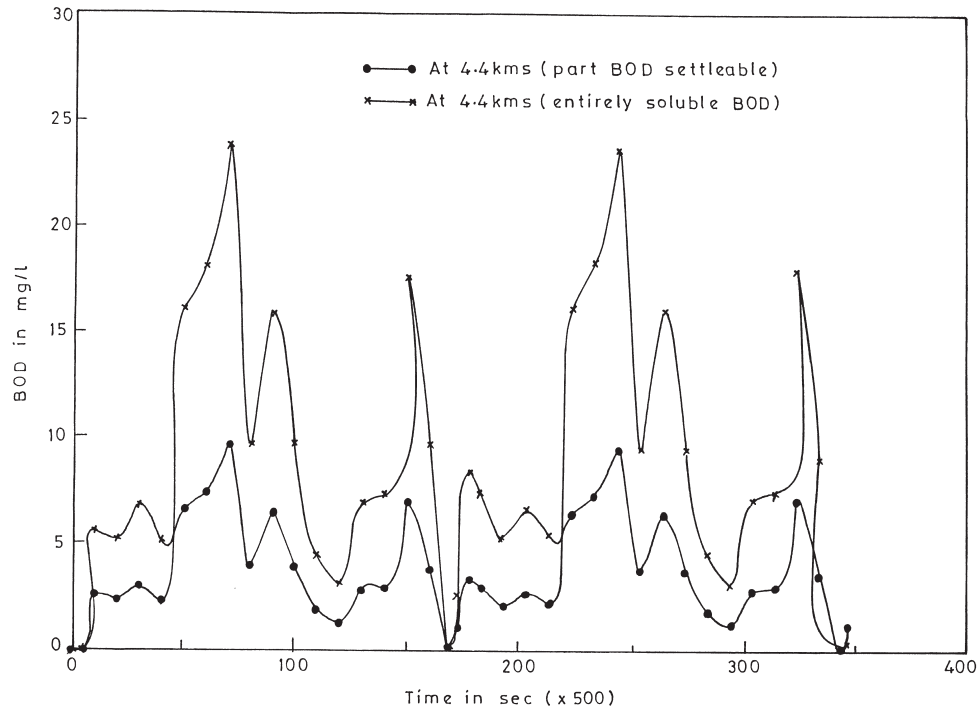


Fig. 4. Comparison of distribution of total BOD for the case when part BOD is settleable with the case of entirely soluble BOD at 4.4 km (inside the sedimentation region).

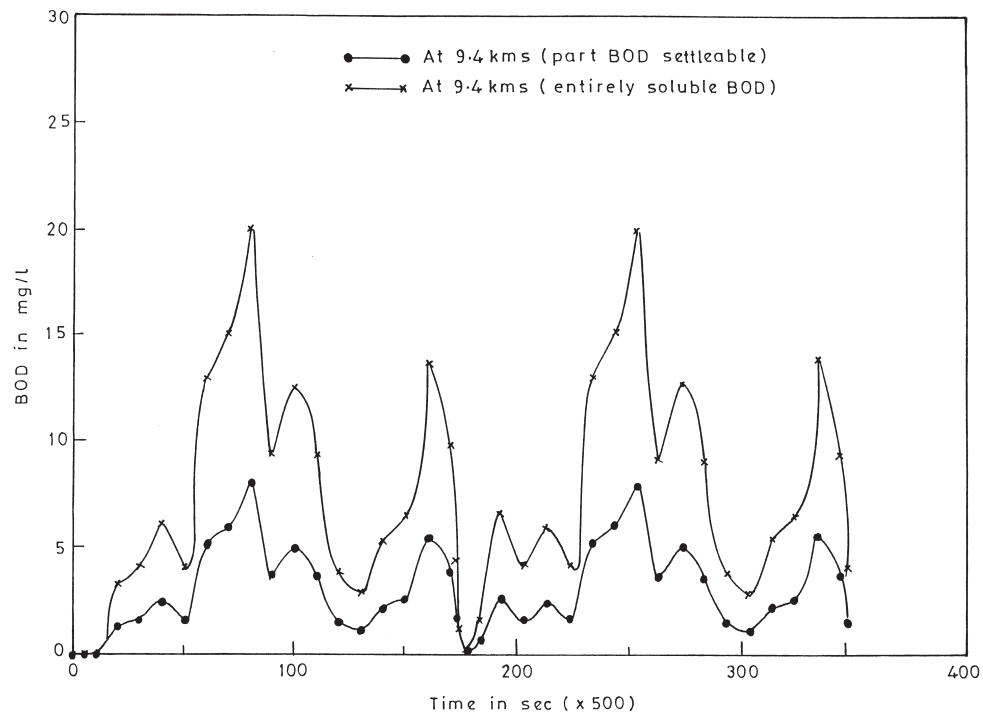


Fig. 5. Comparison of distribution of total BOD for the case when part BOD is settleable with the case of entirely soluble BOD at 9.4 km (outside the sedimentation region).

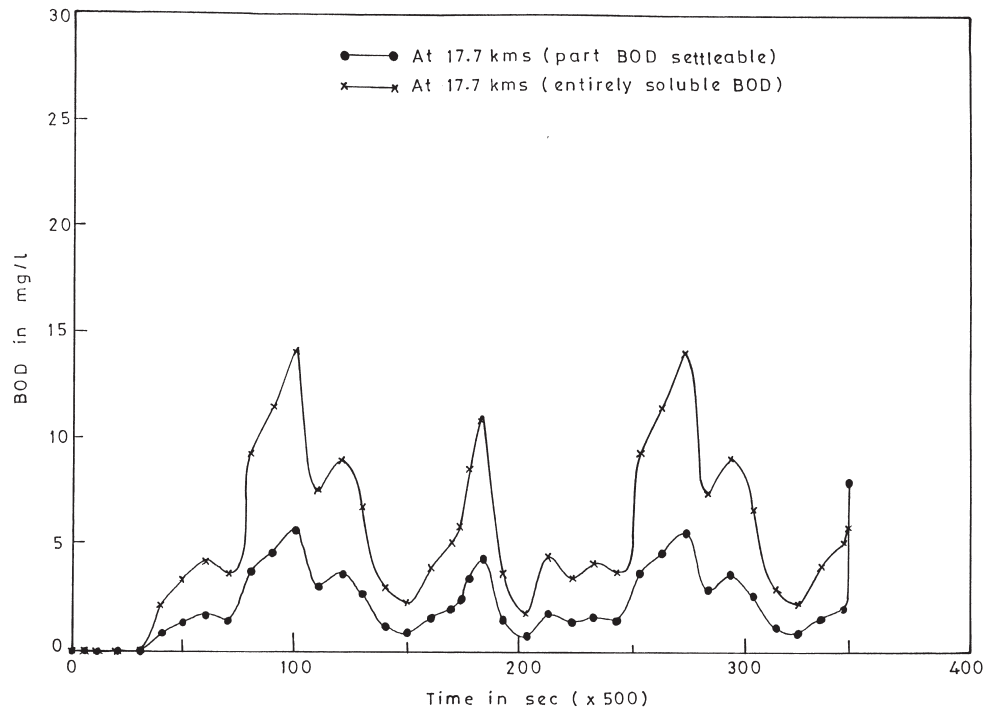


Fig. 6. Comparison of distribution of total BOD for the case when part BOD is settleable with the case of entirely soluble BOD at 17.7 km (outside the sedimentation region).

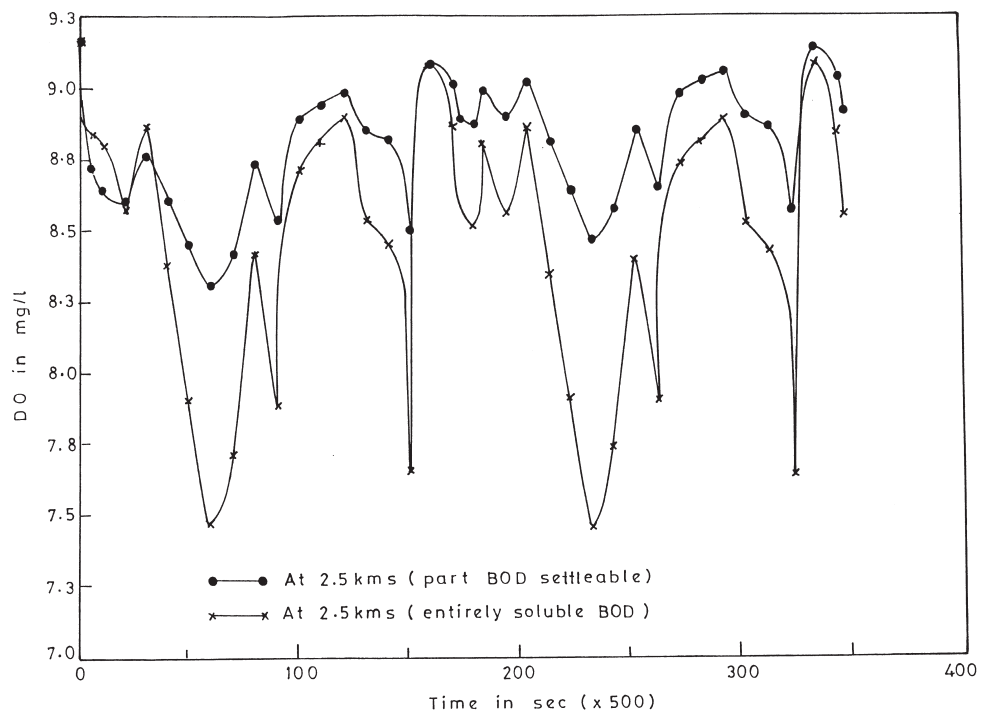


Fig. 7. Comparison of DO distribution for the case when part BOD is settleable with the case of entirely soluble BOD at 2.5 km (inside the sedimentation region).



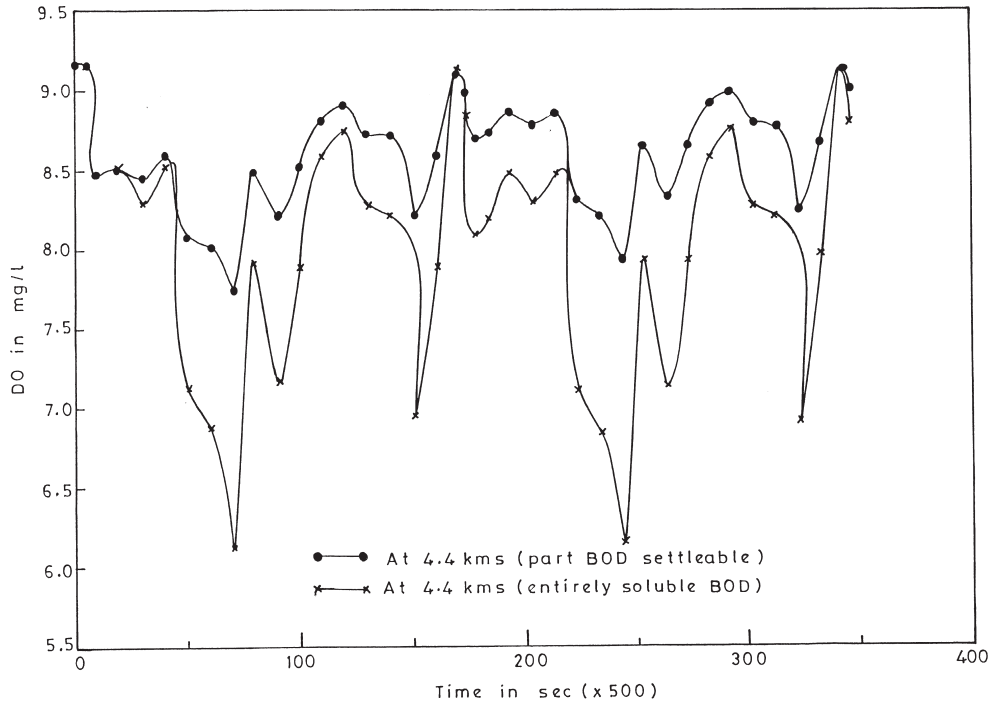


Fig. 8. Comparison of DO distribution for the case when part BOD is settleable with the case of entirely soluble BOD at 4.4 km (outside the sedimentation region).

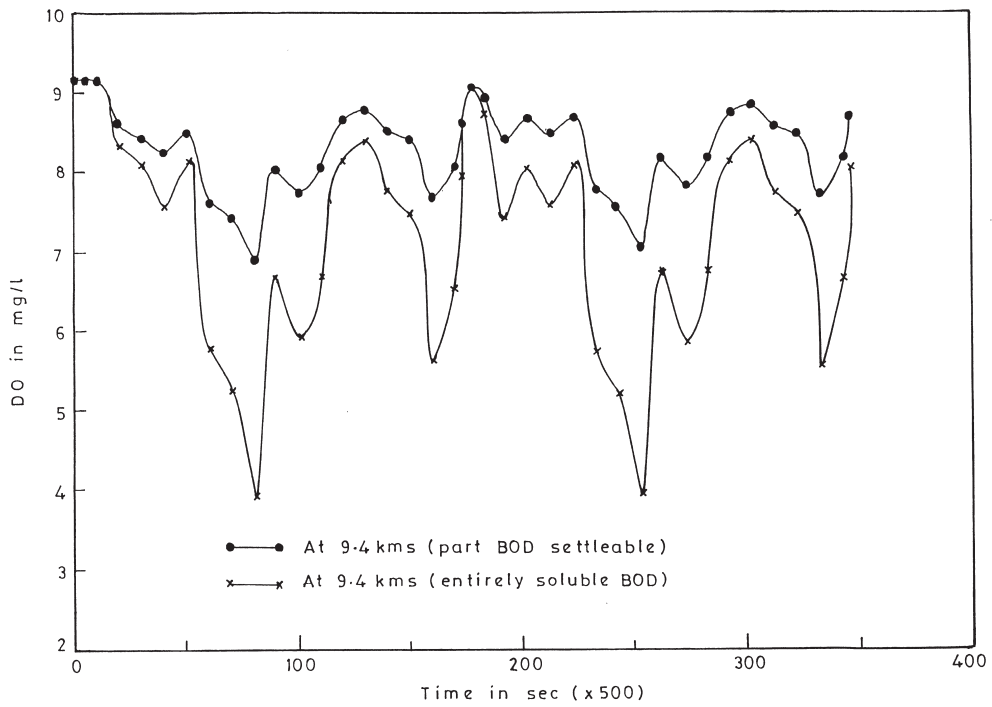


Fig. 9. Comparison of DO distribution for the case when part BOD is settleable with the case of entirely soluble BOD at 9.4 km (outside the sedimentation region).

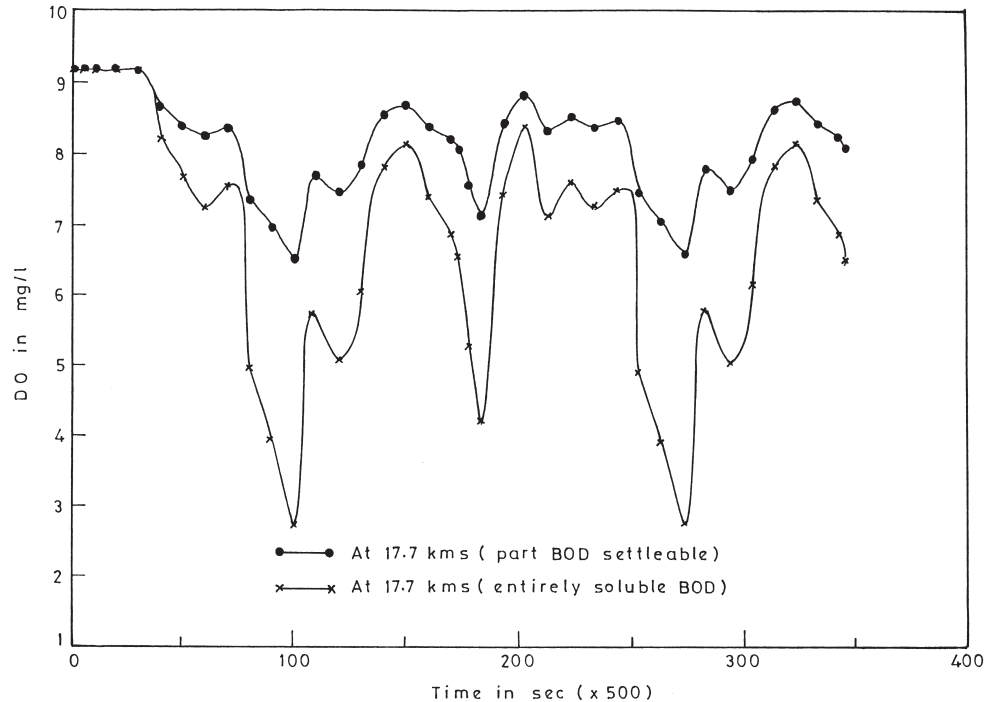


Fig. 10. Comparison of DO distribution for the case when part BOD is settleable with the case of entirely soluble BOD at 17.7 km (outside the sedimentation region).

The reverse process is happening in Figs. 7–9 which show that where the demand is higher the DO concentration would be lower, all other factors affecting stream DO being the same.

It has been observed from Figs. 7–10 that the DO distribution is periodic with the same period as that of the BOD source. However, the amplitude decreases with distance downstream.

It is, therefore, concluded that the conditions of DO in the stream depend on the form and nature of the variation of BOD strength in the waste that is being discharged into the stream.

## 6. Practical application

The model presented in this paper describes the actual situation of domestic sewage discharge into the river through sewers. The model can be applied to predict the DO conditions of the river due to a discharge in which the strength of BOD is varying periodically at the outfall point. To monitor the effects of periodic flows and/or diffusion–dispersion processes, is not an easy task.

The presented model was developed to account for these features and therefore, the predictions would reflect a more rational quality profile of the river. Furthermore, the models can be exploited for predicting the stream's health in real-life situations where some portion (to be established from analytical tests) of the total BOD inputs are bound to be in a settleable form. This important

aspect was totally ignored thus far (Bhargava, 1983, 1986b).

## 7. Conclusion

The combined effect of settleable and dissolved parts of BOD, on DO is shown by the use of a mathematical model. A hypothesized but rational data is used to show the effect of periodically varying and part settleable BOD on the DO, due to the nonavailability of field data. It is concluded that the sedimentation must be taken into account along with dispersion when partially treated or untreated waste invariably enters the stream. The prediction made by the presented model would be more accurate and rational in real-life situations.

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