Optimum design of alternate and conventional furrow fertigation to minimize nitrate loss

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Abstract

Alternate furrow fertigation has shown potential to improve water and fertilizer application efficiency in irrigated areas. The combination of simulation and optimization approaches permits to identify optimum design and management practices in furrow fertigation, resulting in optimum cost, irrigation performance or environmental impact. The objective of this paper is to apply 1D surface and 2D subsurface simulation-optimization models to the minimization of nitrate losses in two types of alternate furrow fertigation: a) variable alternate furrow irrigation; and b) fixed alternate furrow irrigation. For comparison purposes, optimizations are also reported for conventional furrow irrigation. The model uses numerical surface fertigation and soil water models to simulate water flow and nitrate transport in the soil surface and subsurface, respectively. A genetic algorithm is used to solve the optimization problem. Four decision variables (inflow discharge, cutoff time, start time and duration of fertilizer solution injection) were optimized to minimize the selected objective function (nitrate loss) for two fertigation events performed during a maize growing season. The simulation-optimization model succeeded in substantially reducing the value of the objective function, as compared to the field conditions for all irrigation treatments. In the experimental conditions, optimization led to decreased inflow discharge and fertilizer injection during the first half of the irrigation event. This was due to the high potential of the field experiment to lose water and nitrate via runoff. In the optimum conditions, alternate furrow fertigation strongly reduced water and nitrate losses compared to conventional furrow irrigation. The simulation-optimization model stands as a valuable tool for the alleviation of the environmental impact of furrow irrigation.

Key words: alternate furrow irrigation; fertigation; nitrate; optimization; simulation
1. Introduction

Agriculture, as a non-point source polluter, is one of the most important sources of water pollution because of the current abuse of fertilizers and the high losses of irrigation water (IEEP 2000). Nitrate can be easily transported in surface and subsurface water, polluting both surface and groundwater (Ongley 1996). Surface fertigation can be optimized to reduce fertilizer loss and to improve fertilizer distribution uniformity (Abbasi et al. 2003; Adamsen et al. 2005). Alternate furrow irrigation has consistently shown potential to conserve water and to improve water productivity (Kang et al. 2000; Thind et al. 2010; Slatni et al. 2011). Therefore, using fertigation in alternate furrow irrigation could not only conserve water, but also reduce fertilizer losses.

Using simulation models, different scenarios can be evaluated with minimum time and cost to find convenient values of surface irrigation variables, such as inlet discharge and cutoff time. Field experiments are costly and time consuming, and can not explore all values of the relevant irrigation variables. Feinerman and Faakovitzo (1997) developed and applied a mathematical model for identifying optimal scheduling of corn fertilization and irrigation, resulting in maximum farmer’s economic profit. These authors found that leaching was much more sensitive to changes in fertilizer price than to changes in taxes on leached nitrogen. Sabillon and Merkley (2004) developed a mathematical model of furrow fertigation. After validating the model, they executed the model for about 50,000 times to identify the start and end times of fertilizer injection leading to optimum fertilizer application efficiency and uniformity. In their experimental conditions, the best injection duration ranged from 5 to 15 % of cutoff time.
The relationship between surface irrigation and fertigation on one hand, and water and solute transfer on the other, has been analyzed since the turn of the century. Popova et al. (2005) reported the use of subsurface flow model HYDRUS-2D (Šimůnek et al. 1999) for optimization of joint irrigation and fertilization practices in different climates and soil contexts. Abbasi et al. (2004) and Crevoisier et al. (2008) proved that HYDRUS-2D could successfully simulate water and solute transfer for conventional and alternate furrow irrigation. Crevoisier et al. (2008) indicated that HYDRUS-2D performance was better than HYDRUS-1D (Šimůnek et al. 1998) for simulating water content, nitrate concentration and drainage. Zerihun et al. (2005) coupled a surface solute transport model with a subsurface solute transport model (HYDRUS-1D) for simulating surface fertigation in borders and basins. Adequate agreement was reported between field observed and model predicted solute breakthrough curves in the surface stream. Wöhling and Schmitz (2007) also presented a seasonal furrow irrigation model by coupling a 1D surface flow model (zero-inertia), HYDRUS-2D and a crop growth model. The coupled model could adequately predict advance and recession times, soil moisture and crop yield (Wöhling and Mailhol, 2007).

Optimization methods, such as genetic algorithms (Goldberg, 1989), have proven useful for optimizing design and management of irrigation systems for different purposes (economical and environmental, among others). Genetic algorithms (GAs) have been used in the past decade for irrigation project planning (Kuo et al. 2000), off-farm irrigation scheduling (Nixon et al. 2001), flow and water quality predictions in watersheds (Preis and Ostfeld 2008) and for optimizing the cost of localized irrigation projects (Pais et al. 2010). Navabian et al. (2010) presented a 1D model for optimizing fertigation in conventional
furrow irrigation. These authors found that optimization results depended on soil status (bare vs. cropped). This approach could be effectively extended to the optimum design and management of fertigation systems.

Alternate furrow fertigation has proved to have high potential to reduce water and fertilizer losses. Simulating and optimizing the design and management of furrow fertigation will therefore contribute to minimize the environmental pressure of agricultural irrigation on water resources. Thus, the main objective of this study was to develop a 1D surface and 2D subsurface simulation-optimization model for furrow fertigation. The model was applied to two types of alternate furrow irrigation: a) variable alternate furrow irrigation (AFI); and b) fixed alternate furrow irrigation (FFI), as well as for conventional furrow irrigation (CFI). In all cases, the goal was to minimize nitrate losses. Optimization results were compared with experimental results.
2. Material and methods

2.1. Simulation of water and fertilizer flow

Furrow fertigation involves the overland transport of water and fertilizer, and the vertical transfer of part of the water and fertilizer into the soil through the process of infiltration. Fertilizer losses can happen via runoff or via deep percolation, if the fertilizer infiltrates beyond the root zone. In this study, two numerical models were used to simulate a) surface water flow and nitrate transport using a 1D surface fertigation model (Abbasi et al. 2003); and b) subsurface water flow and nitrate transport using a 2D soil water and solute transport model, SWMS-2D (Šimůnek et al. 1994). The description of both models follows.

2.1.1. Surface fertigation

A combined overland water flow and solute transport model was used for simulation of surface fertigation (Abbasi et al. 2003). The governing equations for water flow were solved in the form of a zero-inertia model of the Saint-Venant’s equations:

\[
\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + \frac{\partial z}{\partial t} = 0
\]  

\[
\frac{\partial y}{\partial x} = S_0 - S_f
\]

where \( Q \) is flow rate \([L^3 T^{-1}]\); \( A \) is flow area \([L^2]\); \( z \) is infiltrated water volume per unit length of the field \([L^3 L^{-1}]\); \( y \) is flow depth \([L]\); \( S_0 \) is field slope (dimensionless); \( S_f \) is hydraulic resistance slope (dimensionless); and \( t \) and \( x \) are time \([T]\) and space \([L]\), respectively. Infiltration was characterized using the Kostiakov-Lewis equation:
\[ z = k \tau^a + f_0 \tau \]  

where \( \tau \) is the opportunity time [T], \( a \) (dimensionless), \( k \) [L\(^2\) T\(^{-a}\)], and \( f_0 \) [L\(^2\) T\(^{-1}\)] are the infiltration parameters of the Kostiakov-Lewis equation.

Solute transport was modeled using the 1D cross-sectional average dispersion equation (Cunge et al. 1980):

\[ \frac{\partial(A\overline{C})}{\partial t} + \frac{\partial(A\overline{UC})}{\partial x} = \frac{\partial}{\partial x} \left( AK_x \frac{\partial C}{\partial x} \right) \]  

where \( C \) and \( U \) are cross-sectional average concentration [M L\(^{-3}\)] and velocity [L T\(^{-1}\)], respectively; and \( K_x \) is the longitudinal dispersion coefficient [L\(^2\) T\(^{-1}\)]. Coefficient \( K_x \) incorporates both dispersion due to differential advection and turbulent diffusion (Cunge et al. 1980). The dispersion coefficient for transport in overland flow can be described as:

\[ K_x = D_x U_x + D_d \]  

where \( D_x \) is longitudinal dispersivity [L]; \( D_d \) is molecular diffusion in free water [L\(^2\) T\(^{-1}\)], and \( U_x \) is overland flow velocity at location \( x \) [L T\(^{-1}\)].

Model solutions permit to obtain the distribution along the furrow of infiltrated water and nitrate. These values can be used to determine \( C U_w \) and \( C U_n \), the Christiansen Uniformity Coefficients for water and nitrate, respectively.

Model input data include furrow geometry, infiltration, roughness, discharge, and solute properties. The upstream boundary condition is the irrigation discharge for water and the applied nitrate concentration for fertilizer. The downstream boundary condition usually includes uniform runoff flow or blocked-end runoff for water, and zero concentration
gradient for fertilizer. Zero flow depth, velocity and fertilizer concentration are used as initial conditions along the entire furrow. Model output includes water runoff ratio, nitrate concentration and mass in runoff and the uniformity coefficients of water and nitrate. The validation of this model using field experiments (Ebrahimian et al. 2011) indicated that the model could successfully simulate surface fertigation in conventional and alternate furrows.

### 2.1.2. SWMS-2D

The 2D water and solute transport model SWMS-2D was applied to the simulation of water and nitrate transfer in the soil. A modified form of the Richards' equation governs the flow of water in the soil:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left[ K \left( K_{ij} \frac{\partial h}{\partial x_j} + K_e \right) \right] - S
\]  

(6)

where \( \theta \) is the volumetric water content (dimensionless), \( h \) is the pressure head [L], \( S \) is a sink term [T\(^{-1}\)], \( x_i \) and \( x_j \) are the spatial coordinates [L], \( t \) is time [T], \( K_{ij} \) are components of a dimensionless anisotropy tensor \( K^A \), and \( K \) is the unsaturated hydraulic conductivity function [L T\(^{-1}\)].

Nitrate transfer within the soil was simulated by solving the following version of the advection–dispersion equation, which takes into account the transformation of ammonium into nitrate:

\[
\frac{\partial c}{\partial t} = \frac{\partial}{\partial x_i} \left( D_{ij} \frac{\partial c}{\partial x_j} \right) - \frac{\partial q_i c}{\partial x_i} + \gamma_w \theta - Sc_i
\]  

(7)

where \( c \) is the nitrate concentration in the soil [M L\(^{-3}\)], \( q_i \) is the i-th component of the volumetric flux [L T\(^{-1}\)], \( D_{ij} \) is the dispersion coefficient tensor [L\(^2\) T\(^{-1}\)], \( \gamma_w \) is the zero-order
rate constant for nitrate production by ammonium degradation in the soil solution [M L$^{-3}$ T$^{-1}$], $S$ is the sink term of the water flow in the Richards’ equation, and $c_s$ is the concentration of the sink term [M L$^{-3}$]. The Galerkin finite element method was used to solve this equation, subjected to appropriate initial and boundary conditions.

Water/nitrate deep percolation was estimated by SWMS-2D as the percentage of the applied water/nitrate percolating below the root zone in a certain time. The flow domain corresponding to CFI consisted of the wet furrow and the furrow ridge. In the alternate furrow treatments, the simulation domain included the dry and wet furrows and their respective furrow ridges. The layout and simulation geometry and boundary conditions for conventional and alternate furrow irrigation is presented in Fig. 1.

Combination of simulated water/nitrate runoff and deep percolation permits to estimate the water and nitrate efficiency following an irrigation event. Water and nitrate runoff ($R_{OW}$ and $R_{ON}$) and deep percolation ($D_{PW}$ and $D_{PN}$) can be estimated as the ratio between the lost and applied nitrate and water. This permits to obtain an estimate of the efficiency associated to water and nitrate application ($E_w$ and $E_n$, respectively).

$$E = 1 - (D_P + R_O)$$

(8)

2.2. Optimization

Reducing the mass of exported pollutants is the key to reduce the negative environmental off-site effects of irrigation. Optimizing water and solute transport can maximize water and fertilizer productivity and also minimize water pollution resulting from agrochemicals. The
optimization procedure described in this section was designed to minimize nitrate load in
the return flows of furrow fertigated fields.

2.2.1. Objective function

The following objective function was used to minimize nitrate losses in alternate and
canventional furrow fertigation:

\[
OF = \sum_{i=0}^{T \text{irr}} M_{di} + \sum_{i=0}^{T \text{cutoff}} M_{ri} = M_{dp} + M_{ro}
\]  

(9)

where \( OF \) (g) is the objective function; \( M_{di} \) (g) and \( M_{ri} \) (g) are nitrate mass in deep
percolation and runoff, respectively; \( T \) (min) and \( T_{\text{cutoff}} \) (min) are irrigation interval and cutoff
time, respectively; \( M_{dp} \) (g) and \( M_{ro} \) (g) are total nitrate mass in deep percolation and runoff,
respectively. Parameters corresponding to deep percolation and runoff were predicted using
SWMS-2D and the surface fertigation model, respectively.

2.2.2. Decision variables and constraints

According to Zerihun et al. (1996), inflow discharge, cutoff time, infiltration parameters
and furrow geometry and slope have a significant effect on the production of runoff and
deep percolation. In this study, inflow discharge and cutoff time were chosen as the
irrigation decision variables to be optimized. It is quite simple for farmers to modify these
variables, as compared to modifying soil characteristics and field geometry. Furthermore,
studies by Sanchez and Zerihun (2002) and Smith et al. (2007) have shown that the start
time, the duration of fertigation and the application method of the fertilizer solution (pulsed
vs. continuous) affect fertilizer losses in furrow irrigation. To simplify the experimental
procedures, fertilizer solutions were applied at a constant rate during each fertigation. The following additional constraints involving the decision variables were further considered in order to obtain sensible and practical results:

\[
q_{\min} \leq q \leq q_{\max} \quad (10)
\]

\[
t_{\min} \leq t_{co} \leq t_{\max} \quad (11)
\]

\[
t_s + t_d \leq t_{co} \quad (12)
\]

\[
E_w \geq 0.4 \quad (13)
\]

\[
CU_w \text{ and } CU_n \geq 0.8 \quad (14)
\]

where \(q\), \(t_{co}\), \(t_s\) and \(t_d\) are inflow discharge (L/s), cutoff time (min) and start time (min) and duration (min) of fertilizer solution injection. \(q_{\min}\) and \(q_{\max}\) are minimum and maximum inflow discharge (L/s), respectively. \(t_{\min}\) and \(t_{\max}\) are minimum and maximum cutoff time (min), respectively.

The maximum admissible inflow discharge \(q_{\max}\) was calculated by the following simple empirical function (Booher 1976):

\[
q_{\max} = \frac{0.6}{S} \quad (15)
\]

where \(S\) is furrow slope (%). Reddy and Apolayo (1991), and Navabian et al. (2010) chose 10 and 15% of the maximum (non-erosive) inflow discharge as minimum inflow discharge,
respectively. In this research, the minimum inflow discharge ($q_{\text{min}}$) was assumed to be 10% of $q_{\text{max}}$.

The minimum cutoff time ($t_{\text{min}}$) was based on full irrigation at the end of the furrow, and was calculated as the summation of net opportunity time for target application depth ($t_{\text{req}}$) and total advance time ($t_l$).

\[ t_{\text{min}} = t_{\text{req}} + t_l \]  

(16)

The above expression was originally designed for open-end furrow systems. This procedure neglects the duration of the depletion phase. This assumption is sensible for short and steep furrows. However, considerable additional infiltration might take place in long and level furrows.

The maximum cutoff time was approximated as follows:

\[ t_{\text{max}} = t_{\text{min}} + 2t_{\text{req}} \]  

(17)

Restrictions above can be modified at the discretion of the user of the reported methodology, responding to actual field conditions or specific interests.

2.2.3. Genetic algorithms

A genetic algorithm (GA) is a search/optimization technique based on reproducing the mechanisms of natural selection. In this technique, successive generations evolve and generate more fit individuals based on Darwinian survival of the fittest. As previously stated, GA has been effectively applied for different areas of water and irrigation issues. In
this paper the GA technique was used to develop a simulation-optimization model for furrow fertigation. Among the different optimization techniques available for the solution of this optimization problem, the GA technique stands as an adequate approach to this problem characterized by a relatively large number of optimization variables and an unknown error surface. Once the outcome of the optimization process is assessed in this paper, the efficiency and convenience of alternative optimization methods could be specifically assessed.

The basic operations of the genetic algorithm are described in this paragraph following Praveen et al. (2006). First, the decision variables are encoded into a binary form called a “chromosome” because it gives the genetic encoding (genes or bits) describing each potential solution. Next, an initial population of potential solutions is created, usually by filling a set of chromosomes (population members) with random initial values. Each member of the population is then evaluated to assess how well it performs with respect to the user-specified objective function and constraints (fitness or objective function). Then the population is transformed into a new population (the next generation) using three primary operations: selection, crossover, and mutation. A fourth operator, elitism, is also usually included to ensure that good solutions are not lost from one generation to the next. This transformation process from one generation to the next continues until the population converges to the optimal solution.

The Carroll FORTRAN GA (Carroll 1996) is a computer simulation of such evolution where the user provides the environment (function) in which the population must evolve. This software release includes conventional GA concepts in addition to jump/creep mutations, uniform crossover, niching and elitism. The scheme used in this research was
“tournament selection”, with a shuffling technique for randomly selecting pairs for mating.

This program initializes a random sample of individuals with different parameters to be optimized using the genetic algorithm approach. In order to obtain fast convergence and a global optimum value, it is important to choose adequate values of the population size, the number of generations and the crossover and mutation probabilities. The respective values of these parameters were set to 200, 200, 0.5 and 0.01, respectively, following Carroll (1996) and Praveen et al. (2006).

2.3. Model development

The simulation-optimization model includes six subprograms: 1. Determination of cutoff time; 2. Surface fertigation simulation; 3. Preparation of input files for SWMS-2D; 4. SWMS-2D simulation; 5. Determination of water and nitrate losses in deep percolation; and 6. Genetic algorithm. All these subprograms were written in the FORTRAN programming language. The first, third and fifth subprograms are discussed in the following sections. Finally, the general information flow in the optimization process is discussed.

2.3.1. Cutoff time

This subprogram was developed to determine the minimum and maximum values of the cutoff time (Eqs. 16 and 17). The cutoff time was calculated following Walker and Skogerboe (1987):

\[ t_{co} = t_{dc} - \frac{A_t L}{2Q_o} \] 

(18)
where $t_{eo}$ and $t_{de}$ are the cutoff and depletion times (min), respectively; $A_0$ is the flow cross-section in the furrow inlet (m$^2$); $L$ is the furrow length (m) and $Q_0$ is the inflow discharge (m$^3$/min). $t_{de}$ is iteratively calculated using the following equation:

$$\begin{align*}
(t_{de})_{i+1} &= t_r - \frac{0.095n^{0.47565}S_y^{0.20735}L^{-0.6829}}{f^{0.52435}S_0^{0.237825}} \\
\text{Where}
\end{align*}$$

$$t_r = \tau_{req} + t_l$$

$$I = \frac{ak}{2} \left[ \left( t_{de} \right)_{i}^{(a-1)} + \left( t_{de} - t_l \right)^{(a-1)} \right] + f_0$$

$$S_y = \left( \frac{(Q_0 - IL)n}{60S_0} \right)^{0.6}$$

where $n$ is the Manning roughness coefficient (m$^{1/6}$); $S_0$ is the furrow slope; $\tau_{req}$ is the net opportunity time for target application depth (min); $t_l$ is the advance time (min); and $I$ is the infiltration rate (m/min). The initial value for $t_{de}$ is assumed to be equal to $t_r$. $t_l$ is determined solving the implicit water balance equation using the Newton-Raphson method:

$$Q_0 t_x = \sigma_y A_0 x + \sigma_z k t_x^{a} x + \frac{1}{(1+r)} f_0 t_x x$$

where $t_x$ is the time for advancing water to distance $x$ from the furrow inlet (min); $\sigma_y$ and $\sigma_z$ are the surface and subsurface shape factors, respectively, and $r$ is the exponent of the advance equation ($x = pt^r$, $p$ is an empirical coefficient) (Walker and Skogerboe 1987).
2.3.2. Generating input files for SWMS-2D

The SWMS-2D model needs three input files for simulating water flow and solute transport: "selector.in", "grid.in" and "atmosph.in". The "selector.in" file contains the soil water retention curve, the number of soil layers, plant uptake and solute transport parameters. The flow domain geometry, the initial values of soil water and nitrate content and the boundary conditions are stated in the "grid.in" file. Evaporation, transpiration, rainfall, nitrate concentration of irrigation water, start time and duration of fertilizer solution injection, cutoff time, irrigation interval and water depth/infiltration rate in furrow make part of the "atmosph.in" file. Both "selector.in" and "grid.in" files are independent of the decision variables used in this application. This is not the case of the "atmosph.in" file, whose values are updated during the optimization process. Therefore this subprogram generates a new "atmosph.in" file each time the decision variables are updated by the genetic algorithm. The subprogram generates this file for the upstream, middle and downstream furrow sections, in accordance with the advance and recession times. Soil water and solute flow in each furrow were simulated at these three sections, in an effort to characterize the effect of irrigation variability on the soil.

2.3.3. Water and nitrate losses in deep percolation

The average value of water and nitrate losses to deep percolation along the furrow was used for calculating the objective function. This subprogram used SWMS-2D output. The average nitrate mass in deep percolation per unit length \((M)\) was calculated as follows:

\[
M = \frac{M_u + M_m + M_d}{3} \quad (24)
\]
where $M_u$, $M_m$ and $M_d$ are the nitrate masses in deep percolation per unit length (g/m) at the upstream, middle and downstream of the field, respectively. The total mass of nitrate leached from the root zone ($M_{dp}$) for the entire field was determined multiplying $M$ times the furrow length ($L$).

### 2.3.4. Optimization process

The different simulation models were linked to the genetic algorithm in order to optimize the decision variables ($q$, $t_{co}$, $t_s$ and $t_d$), by minimizing the objective function. The optimum set of decision variables must satisfy all constraints while minimizing nitrate losses.

The flowchart of the simulation-optimization model is presented in Fig. 2. First, the initial population (containing values of the decision variables for each individual) is generated. The simulation models are executed for each individual and the values of the objective function are determined. The convergence criterion (the number of generations) is checked. If this criterion is satisfied the model stops. Otherwise, three genetic algorithm operators (selection, crossover and mutation) are executed to produce a new generation (characterized by new individual values of the decision variables).

The model was run in a cluster of 28 high-performance processors using the Linux operative system. The cluster was located at the Fluid Mechanics Area of the University of Zaragoza. The processing speed of each processor was 2.80 GHz. Consequently, the compound processing speed of the cluster was 78.4 GHz. The code was parallelized to exploit the computing power of the cluster and to reduce the computational time.
Six model runs were performed (three irrigation treatments times two fertigation events). Each run explored 40,000 different sets of values of the decision variables (the population size multiplied by the number of generations). If the set of decision variables satisfied the constraints, the SWMS-2D and surface fertigation models were run three times and one time, respectively. In each of the cluster processors, the SWMS and surface fertigation models required execution times of 10-120 s, respectively, depending on the values of the decision variables and on the irrigation treatment. Computational time was larger for alternate furrow irrigation than for conventional furrow irrigation, owing to the flow domain requirements in SWMS-2D. A complete model run took about 1-2 weeks. The current execution time negatively affects the applicability of the proposed software development. This problem needs to be addressed by simplifications in the simulation and optimization approaches and by improvements in computational speed. It seems clear that most of the improvements in the short-term will come from the identification of conceptual simplifications showing moderate effect on model performance.

### 2.4. Field experiment

A field experiment was carried out at the Experimental Station of the College of Agriculture and Natural Resources, University of Tehran, Karaj in 2010. The purpose of this experiment was to collect field data on alternate furrow fertigation in order to calibrate the simulation models used in this research. Ebrahimian et al. (2012) presented this experiment in detail, and disseminated the experimental database. A brief description of the experimental conditions follows.
2.4.1. The experimental setup

As previously discussed, the experiment involved three irrigation treatments: variable alternate furrow irrigation (AFI), fixed alternate furrow irrigation (FFI), and conventional furrow irrigation (CFI). Fertigation was designed to satisfy the water and nutrient needs of maize production when applied to all furrows in the field. Pre-sowing fertilizer application was limited to 10% of the crop’s nitrogen fertilizer requirements (200 kg N ha⁻¹), applied a day before sowing using a mechanical broadcaster. Three nitrogen dressings (each one amounting to 30% of the fertilizer requirements) were applied at the vegetative (seven leaves, in July 7), flowering (August 9) and grain filling (August 30) stages using surface fertigation. Nitrogen fertilizer was applied in the form of granulated ammonium nitrate. The same amount of water and fertilizer was applied to all irrigated furrows. Thus, the water and fertilizer application rate per unit area were twice as much for conventional irrigation than for the two alternate irrigation treatments.

The average soil physical and chemical characteristics are presented in Table 1. Soil depth was limited to 0.60 m due to the presence of a gravel layer. In total, 14 furrows were established in this experimental study (3, 5, and 6 furrows for the CFI, FFI, and AFI treatments, respectively). Furrow spacing was 0.75 m, furrow length was 86 m, and the longitudinal slope was 0.0093. Water samples at the furrows’ inflow and outflow were used to determine the time evolution of nitrate concentration. Auger soil samples were collected at the dry (non-irrigated) and wet (irrigated) furrow beds and ridges in three soil layers (0.0-0.2, 0.2-0.4 and 0.4-0.6 m). Soil water content and nitrate concentration were determined in the soil samples before and after the fertigation events. Irrigation was applied on a 7 day
interval throughout the irrigation season. During the first fertigation event, discharge was 0.262 L/s, and cutoff time was 240 min. In this event, fertilizer injection started at the completion of the advance phase (at a time of about 50 min, depending on the particular furrow), and lasted for 150 min. During the second fertigation event, discharge was 0.388 L/s, and irrigation cutoff time was 360 min. In this event the fertilizer solution was injected during the first half of the irrigation time (180 min injection time).

2.4.2. Estimating furrow infiltration

The parameters of a Kostiakov-Lewis infiltration equation were separately estimated for all irrigation treatments in each fertigation event using the two-point method (Elliott and Walker 1982). These parameters were used to calibrate the surface fertigation model. Accuracy in the determination of advance data and basic infiltration rate (steady-infiltration rate) led to an adequate estimation of furrow infiltration. This was evidenced by the low relative error between measured and estimated infiltrated volume: below 4% in all irrigation treatments and fertigation events.

2.4.3. Simulating fertilizer transport and transformation

During SWMS-2D model calibration, the water flow and nitrate transport parameters were estimated by inverse solution, using the Levenberg–Marquardt optimization module in the HYDRUS-2D software (Šimůnek et al. 1999). The values of the estimated parameters resulted in minimum error between observed and simulated values. The SWMS-2D model was separately calibrated at three furrow sections (upstream, middle and downstream), and for each irrigation treatment using the calibrated parameters estimated by the inverse solution of HYDRUS-2D. The method for calibrating, validating and defining
initial/boundary conditions of HYDRUS-2D in the specific conditions of this problem was presented by Ebrahimian et al. (2011). These authors reported that HYDRUS-2D performed adequately when applied to conventional and alternate furrow fertigation. The experiments involved the use of ammonium nitrate as a nitrogen fertilizer. Ammonium transport was not simulated in this study, which only simulated nitrate. Soil nitrate concentration measurements and their temporal and spatial changes were used to characterize nitrate generation in the soil through the nitrification process (Ebrahimian et al. 2011). This involved the estimation of parameter $\gamma_w$ in equation 7 through the abovementioned inverse solution procedure.

Although the surface fertigation and SWMS-2D models were separately run and calibrated, infiltration calculated with the extended Kostiakov equation resulted very similar to SWMS-2D results. This can be illustrated by the simulation results of the second fertigation event. The total estimated infiltrated volume of fixed alternate furrow irrigation were 5.183 and 5.097 m$^3$ for the surface fertigation and SWMS models, respectively. These values resulted very similar to the measured value of 4.927 m$^3$. Similitude between these magnitudes is crucial to the success of the proposed modeling scheme, since infiltration is the process connecting both simulation models.

Nitrate leaching was determined in the experimental furrows taking into consideration a rootzone of 0.60 m and a leaching time of 7 days. This time is equivalent to the experimental irrigation interval.
3. Results and discussions

3.1. Field study

The values of the objective function were calculated using the results of the combined simulation models (without the optimization process) applied to all irrigation treatments and using the experimental values of the decision variables (Table 2). These decision variables resulted in high water and nitrate uniformities (CU > 90 %) and appropriate water application efficiency. Low values of nitrate application efficiency were predicted, particularly for CFI in the second fertigation. Having higher water and nitrate application efficiency, the AFI treatment resulted in lower values of the objective function than the FFI and CFI treatments. The highest water and nitrate losses were due to runoff, as compared to deep percolation. This was particularly true in the second fertigation.

3.2. Optimization

The minimum values of $OF$ for AFI, FFI and CFI were 76.9, 86.5 and 182.6 g in the first fertigation and 87.1, 92.4 and 213.3 g in the second fertigation, respectively (Table 3). CFI caused larger nitrate losses than AFI and FFI. Small differences were found between FFI and AFI. The minimum value of the objective function was substantially lower than the value obtained under field conditions (Tables 2 and 3). Optimization reduced the OF by 77, 81 and 61 % in the first fertigation and by 80, 80 and 68 % in the second fertigation, respect to the experimental values for the AFI, FFI and CFI treatments, respectively. This finding is just an indication of the model’s potential to improve furrow fertigation systems regarding environmental risks on water resources. Results need to be used with caution,
since the experimental conditions were not particularly representative of local fertigation application practices.

In the second fertigation the optimum total inflow volume ($Q_{Tco}$) was higher than it in the first fertigation, due to higher crop water requirements leading to larger soil water depletion. The highest optimum values of the inflow discharge were obtained for the AFI and FFI treatments, due to the high infiltration rate in alternate furrows. Optimum CFI cutoff time was higher than for AFI and FFI in order to compensate for the low values of the inflow discharge. The experimental field showed high potential for water and nitrate runoff losses, particularly for CFI. This was due to the high slope, the fine soil texture and the relatively short length of the experimental furrows. As a consequence, the model identified optimum discharge values which were always lower than the experimental ones.

Comparing the first and second optimum fertigation events, inflow discharge somewhat increased for AFI and FFI, and decreased for CFI. This seems to be related to the differences in infiltration parameters and soil water depletion. The optimum discharges were 38 and 89 % higher in alternate furrows (average of AFI and FFI) than in conventional furrows in the first and second fertigation events, respectively. This trend will need to be confirmed in further research. The opposite trend was observed in the time of cutoff. Combining both variables, the optimum total inflow volume was 12 and 8 % lower in alternate furrows than in conventional furrows in the first and second fertigation events, respectively. Differences between alternate furrow treatments were not relevant.

The field study and the simulation results agree in that the wet furrow bottom received more water and nitrate than the ridge and the dry furrow bottom (Ebrahimian et al. 2011). Soil water and fertilizer distribution were highly affected by the decision variables: runoff
and deep percolation strongly depended on timing of fertilizer application. The optimum value for the start time of fertilizer injection was lower than one third of the cutoff time \((t_s \leq \frac{1}{3} t_{co})\), while the optimum duration of fertilizer injection was less than half of the cutoff time \((t_d \leq \frac{1}{2} t_{co})\) for all irrigation treatments, both fertigation events. The sum of start time and duration of fertilizer injection was in all cases lower than half of the cutoff time (i.e. \(t_s + t_d \leq 0.5 t_{co}\)). Abbasi et al. (2003) also showed that the fertilizer application in the first half of irrigation increased fertilizer application efficiency, while fertilizer application in the second half of irrigation increased fertilizer uniformity for blocked-end and free draining furrows. Playán and Faci (1997) reported that the application of fertilizer during the entire irrigation event often produced maximum uniformity in blocked-end borders and level basins. Sabillon and Merkley (2004) reported that the optimum start time of fertilizer injection happened during the advance time, leading to optimum fertilizer application uniformity and efficiency for relatively steep and long free draining furrows. Navabian et al. (2010) showed that the best start time and duration for fertilizer injection occurred at the beginning of irrigation and at 30% of cutoff time, respectively, for relatively short free draining furrows. In the present study, the simulation-optimization model identified optimum fertilizer injection during the first half of the cutoff time. Early injection permitted effective control of water and nitrate runoff losses. This resulted in increased water and nitrate deep percolation losses because of higher infiltration at the early stages of irrigation. A trade-off process between runoff and deep percolation losses permitted to minimize the objective function. In this particular case, early fertilizer application resulted in optimum results. In the field conditions the simulation model predicted higher nitrate loss to runoff than to deep percolation (Table 2). However, the simulation-optimization model selected values of the decision variables which increased nitrate losses to deep percolation and
strongly reduced runoff losses (Table 3). Nitrate concentration and mass in deep
percolation were larger than in runoff for all irrigation treatments.

Both the simulation and optimization processes further proved that alternate furrow
irrigation can considerably reduce water and nitrate losses compared to conventional
furrow irrigation. Differences in nitrate losses (per furrow) were moderate in the
experimental field conditions, with 16 and 37 % reduction for alternate furrow respect to
conventional furrow irrigation (for the first and second fertigation events, respectively). In
the optimum solution, reductions in nitrate losses for both fertigation events escalated to 55
and 58 %, respectively. Optimization seems to be very important to control nitrate losses.
The relevance of these improvements is magnified by the fact that in alternate furrows the
effective irrigated area per furrow is double than in conventional furrows.

In the optimum conditions the coefficient of uniformity for water \( (CU_w) \) was higher than
\( CU_n \), while nitrate efficiency \( (E_n) \) was larger than water efficiency \( (E_w) \) for all cases. \( E_w \) and
\( E_n \) ranged from 74 to 88 % and from 75 to 91 %, respectively. With respect to the
experimental conditions, optimization slightly decreased water and nitrate uniformity, and
strongly increased water and nitrate efficiency. Optimum alternate furrow irrigation
increased water application efficiency by 10 and 17 % respect to conventional furrow
irrigation (first and second irrigation events, respectively). Regarding nitrate efficiency, the
increase respect to conventional furrow irrigation amounted to 15 and 19 % for both
irrigation events. AFI and FFI showed similar values of \( E_w \) and \( E_n \). For these treatments, \( E_w \)
was higher in the second fertigation than in the first fertigation (87 vs. 81 %).
The generational evolution of the objective function and water and nitrate application efficiency is presented in Fig. 3 for all irrigation treatments and for both fertigation events. The simulation-optimization model showed adequate convergence in all cases (e.g. after 13 generations for AFI in the first fertigation and 97 generations for CFI in the second fertigation). The values of the objective function strongly varied in the first generations of the optimization solution. Gradual variations of the objective function were observed with increasing generations, until \( OF \) converged to constant, final values. These results illustrate that a convergence criterion for problem solution would easily result in very significant reductions in computational time requirements.

The objective function (nitrate losses) and nitrate efficiency showed clear evolutive trends (Fig. 3). However, water application efficiency showed a more erratic pattern during the first generations. This seems to be due to the initial values of inflow discharge and cutoff time. After a few generations, water efficiency increased with increasing generations in a pattern similar to nitrate efficiency.

The model identified optimum decision variables that minimized not only nitrate losses, but also water losses. This is partly due to the high solubility of nitrate in water. As a consequence, nitrate transport highly depends on water flow. The optimum design of nitrate fertigation led to a suboptimal solution from the viewpoint of reducing water losses. When the model was used to maximize irrigation application efficiency in the experimental problem, using discharge and time of cutoff as decision variables, the efficiency of the AFI, FFI and CFI treatments was 83, 82 and 78 % in the first fertigation. These values are very similar to those obtained for the minimization of nitrate losses, with average absolute differences of 2.5 %. This similitude in the results of the different optimization problems is
not always guaranteed, since particular cases may lead to different solutions. The convergence of results for water and nitrate optimization suggests a relevant simplification: solving the optimization problem separately, i.e, optimize the irrigation flow rate and cutoff time first, and then optimize the fertigation strategy. According to the experimental results, fertilizer losses would be minimized if water losses were minimal. Caution should be used when analyzing this simplification, since it is not difficult to devise a case in which minimum water losses result in relevant fertilizer losses (for instance through the runoff of a small volume of water with high fertilizer concentration).

3.3. Optimization with fixed inflow discharge

In furrow irrigation, infiltration parameters have been reported to depend on flow depth, and as a consequence, on inflow discharge (Playán et al. 2004; Rodriguez 2003). While this variation can be neglected for small changes in depth or discharge, the difference between experimental and optimum discharges reported in the previous section results relevant. As a consequence, results above should be interpreted with caution. An infiltration equation using flow depth or wetted perimeter as an additional independent variable would have been required to better represent the real world. The experimental complexities evidenced by Playán et al. (2004) situated such analysis out of the scope of the current research.

Taking this limitation in mind, the simulation-optimization model was run again in a more restrictive but more correct study case. The inflow discharge and the time of cutoff were fixed to the experimental values. As a consequence, optimization was only applied to the start time and the duration of fertilizer injection. Results are presented for all irrigation treatments (Table 4). Although the values of the objective function were larger than those
obtained in the previous run (Table 3), the model could successfully decrease nitrate losses. The average reduction respect to the experimental conditions was 50 %. The optimum injection time was relatively short (about 60 min). In agreement with Sabillon and Merkley (2004), the optimum timing for fertilizer injection was the advance time. This simulation run produced higher $CU_n$ and lower $E_n$ than the fully optimized run. Runoff losses considerably increased, particularly for the second fertigation event, due to the use of an inappropriate experimental inflow discharge. Both Zerihun et al. (1996) and Navabian et al. (2010) reported that inflow discharge was the most important parameter conditioning furrow irrigation system performance.

3.4. Minimizing nitrate in deep percolation

Nitrate pollution in runoff and deep percolation are not of equal concern. Polluted runoff water can in many instances be considered as fertilized irrigation water ready for subsequent uses. However, polluted deep percolation water often represents an imminent pollution risk. In order to give proper consideration to these concerns, different weighting factors ($w$) were considered for the deep percolation term in the objective function ($OF=w.M_{dp}+M_{rn}$). The simulation-optimization model was run for the FFI treatment in the second fertigation event for $w=1, 3$ and 5 (Table 5). As expected, the value of the objective function increased with the weighting factor, signaling that any non-unit weighing factor makes $OF$ lose its physical meaning. Increased inflow discharge led to a decrease in infiltration opportunity and, therefore, to decreased deep percolation. However, runoff water and nitrate losses showed large increases. The optimum start time of fertilizer injection was also found during the advance phase. The optimum duration of fertilizer injection was about one third of the cutoff time. Water and nitrate application efficiency
decreased by 20 %. The solution identified in this section could be interesting if runoff losses could be safely re-used in adjacent fields without additional pumping or management costs.
Conclusions

In this study, a simulation-optimization model was developed to optimize the design and management of alternate and conventional furrow fertigation. This approach was based on simulation of 1D surface and 2D subsurface water flow and solute transport. A genetic algorithm was used for minimizing the objective function, based on the mass of nitrate losses. Model results were compared with the output of the simulation models under the field experimental conditions (without optimization). The simulation-optimization model decreased the objective function for all irrigation treatments by an average 74%. Due to high potential to produce runoff in the experimental field, the optimum solution was based on decreasing inflow discharge and increasing the time of cutoff. The optimum fertilizer injection was identified during the advance phase and within the first half of the irrigation time. The model could also minimize both water and nitrate losses for all irrigation treatments with acceptable distribution uniformity. Assuming constant experimental values for inflow discharge and cutoff time, the optimum injection took place in a relatively short time and at a relatively high injection rate.

Both the simulation and simulation-optimization models proved that alternate furrow irrigation (AFI and FFI) could strongly increase water and nitrate application efficiency, as compared to conventional furrow irrigation. AFI showed better performance than FFI for both fertigation events. The model is quite flexible to be applied to many specific tasks regarding surface irrigation and fertigation. Application of this model to furrow fertigation and fertilizer management can effectively minimize water pollution resulting from agricultural activities. However, a significant reduction in computational time will be
required to make this software operational. Research will be required to assess the effect of conceptual simplifications on model building.

**Acknowledgments**

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Tables

Table 1. Physical and chemical soil properties of the experimental field.

Table 2. The values of the objective function and the outputs of the simulation models for field condition

Table 3. Minimum objective function, optimum decision variables and the outputs of the simulation models

Table 4. Minimum objective function, optimum decision variables and the outputs of the simulation models with fixing inflow discharge and cutoff time

Table 5. Minimum objective function and optimum decision variables for the FFI treatment in the second fertigation
Table 1. Physical and chemical soil properties of the experimental field.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Texture</th>
<th>Bulk density (Mg/m³)</th>
<th>FC (-)</th>
<th>PWP (-)</th>
<th>Organic matter (%)</th>
<th>pH</th>
<th>ECₑ (dS/m)</th>
</tr>
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<td>1.51</td>
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<td>OF (g)</td>
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<td>472.3</td>
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<td>t(_{d}) (min)</td>
<td>150.0</td>
<td>150.0</td>
<td>150.0</td>
<td>180.0</td>
<td>180.0</td>
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<tr>
<td>DP (-)</td>
<td>0.056</td>
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<td>CU(_{w}) (-)</td>
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<td>0.940</td>
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<td>0.961</td>
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<td>CU(_{n}) (-)</td>
<td>0.953</td>
<td>0.968</td>
<td>0.939</td>
<td>0.942</td>
<td>0.946</td>
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<tr>
<td>E(_{w}) (-)</td>
<td>0.705</td>
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<td>0.530</td>
<td>0.598</td>
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<td>E(_{n}) (-)</td>
<td>0.603</td>
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<td>0.454</td>
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Table 3. Minimum objective function, optimum decision variables and the outputs of the simulation models

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<th>Variable</th>
<th>First fertigation</th>
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<tr>
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<td>AFI</td>
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<td>$OF \ (g)$</td>
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<td>$q \ (L \ s^{-1})$</td>
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<td>$t_s \ (\text{min})$</td>
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<td>$t_d \ (\text{min})$</td>
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<td>105.7</td>
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<td><strong>Simulation outputs</strong></td>
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<td>$E_n \ (-)$</td>
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Table 4. Minimum objective function, optimum decision variables and the outputs of the simulation models with fixing inflow discharge and cutoff time

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<td>AFI</td>
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<td>$M_{ro}$ (g)</td>
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<td>$E_n$ (-)</td>
<td>0.863</td>
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Table 5. Minimum objective function and optimum decision variables for the FFI treatment in the second fertigation

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<tr>
<th>Weighting factor</th>
<th>OF (g)</th>
<th>q (L s⁻¹)</th>
<th>t_co (min)</th>
<th>t_s (min)</th>
<th>t_d (min)</th>
<th>M_dp (g)</th>
<th>M_ro (g)</th>
<th>E_w (-)</th>
<th>E_n (-)</th>
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<td>1</td>
<td>92.5</td>
<td>0.225</td>
<td>383.4</td>
<td>65.1</td>
<td>121.5</td>
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<td>3</td>
<td>233.5</td>
<td>0.333</td>
<td>322.9</td>
<td>4.9</td>
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<td>5</td>
<td>318.3</td>
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<td>38.5</td>
<td>125.8</td>
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**Figures**

**Fig. 1** a) layout of the furrow irrigation treatments, b) boundary conditions used in SWMS-2D for conventional and alternate furrow irrigation treatments.

**Fig. 2.** Flowchart of the simulation-optimization model

**Fig. 3.** The objective function and water and nitrate efficiency for each generation in the first and second fertigation
Fig. 1 a) layout of the furrow irrigation treatments, b) boundary conditions used in SWMS-2D for conventional and alternate furrow irrigation treatments.
Fig. 2. Flowchart of the simulation-optimization model

1. Generate initial population
2. Call input data
3. Call decision variables
4. Simulation
5. Nitrate mass in runoff
6. Nitrate mass in deep percolation
7. Objective function
8. Convergence criterion
9. Stop
10. Yes
11. Selection operator
12. Crossover operator
13. Mutation operator
14. No
Fig. 3. The objective function and water and nitrate efficiency for each generation in the first and second fertigation.